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Foreign exchange intervention revisited: A new way of estimating censored models

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Abstract

In this paper we investigate what has been a long-standing issue in the international finance literature, namely to capture the behaviour of central banks when deciding over foreign exchange policies. Essentially, the main empirical problem is that a researcher observes numerous and large-scale purchases of foreign currency but a general absence of sales. This asymmetry has motivated the use of heavily dependent parametric models. We take a fresh look at this problem by allowing for a more flexible estimation, robust to various model specifications. Our results indicate that our method outperforms some of the standard models used to date. Hence, our main contribution is to provide policy makers with an improved and readily accessible toolkit to evaluate their actions. To shed some light on this, we estimate policy functions for the cases of Turkey and Colombia and highlight marked differences with the related literature.

1 | INTRODUCTION

Central banks generally conduct foreign exchange operations in order to calm disorderly markets, influence the exchange rate, and to support other market-related transactions (e.g. accumulate or diminish international reserves). Paradoxically, even though there has been an ample strand of empirical work that centers on the effectiveness of central bank intervention, the literature has yet to reach a general consensus.[1] On the one hand, advocates argue that purchases and sales of foreign currency (even if sterilized) can affect the exchange rate by re-balancing portfolio holdings, often assumed to be comprised of imperfectly substitutable assets. Alternatively, critics sustain that central banks cannot simultaneously allow for: (i) free capital flows; (ii) autonomous monetary policy; and (iii) manage the exchange rate. Essentially, questions such as whether a *portfolio channel* exists or whether a *monetary trilemma* prevents the separate use of policies, have motivated some of the most controversial debates in the modern history of central banking.

For the most part, empirical works differ in the way that exogenous variation is identified. Namely, specification issues arise in any model-based approach where a policy function needs to be estimated. Authors may choose different variables to model policy, adopt different functional forms, and treat contextual characteristics in various ways. A caveat, however, is that the use of heavily dependent parametric models sometimes condition the validity of results (see Arabmazar & Schmidt, 1981, 1982).

For example, to control for heteroskedastic errors due to the high frequency nature of exchange rate data, some authors model policy through the use of a Generalized Autoregressive Conditional Heteroskedasticity (GARCH) process.[2] Other studies model policy using a Tobit model in order to capture marked asymmetries between purchases and sales of foreign currency.[3] These asymmetries thus imply the existence of a censoring process; something that in the literature has been coined as a '*fear of floating*' (see Calvo & Reinhart, 2002; Levy-Yeyati & Sturzenegger, 2007).

In this paper we allow for a more flexible estimation of central bank policy functions by using quantile regressions, robust to heteroskedasticity and to any error distribution. Specifically, we follow the methodology presented in de Jong and Herrera (2008), in which a Censored Least Absolute Deviation model (CLAD) is extended to a time-series framework, that is, the authors show asymptotic consistency and normality of the estimators. However, in contrast to de Jong and Herrera (2008), we analyse the small sample properties of the estimators and specific characteristics within the empirical context of foreign exchange intervention. In particular, we test the properties of CLAD estimates for cases in which: (i) the censoring threshold is misspecified; (ii) the degree of censoring varies; (iii) errors are subject to conditional heteroskedasticity; (iv) the distribution of the errors changes; and (v) there are multiple censoring thresholds. We believe that these simulation exercises can provide key insights to future research given that this method has been employed, almost exclusively, using cross-sectional data.

Additionally, we conduct an empirical application of the CLAD methodology by estimating foreign exchange policy functions for the case of two emerging market economies: the case of Turkey and Colombia. We use proprietary data, at a daily frequency, from both the Central Bank of Turkey and the Central Bank of Colombia during the period 2000–2010. We acknowledge that estimating policy functions can be challenging due to a possible simultaneity bias. Namely, central banks can both react to and affect economic variables. Fortunately, similar to Romer and Romer (2004), we observe the exact timing of each intervention episode. The high frequency nature of our data thus allows us to pinpoint the set of information that monetary authorities had at their disposal before and after deciding over policy.[4] Nonetheless, we note that the CLAD methodology alone does not solve endogeneity problems that stem from simultaneity or omitted variable bias. We refer readers to studies like Kim (2003) and Echavarría, Melo-Velandia, and Villamizar-Villegas (2017) for ways to address endogeneity problems when estimating policy functions, either with the use of Structural Vector Auto-Regression (SVAR) models or with an instrumental variable approach.

Our results indicate that the time-series application of CLAD outperforms the Tobit model in terms of bias and root-mean-square-error for cases in which there is conditional heteroskedasticity and different error distributions. Moreover, our proposed method seems to be more robust than the Tobit model when there is misspecification of the censoring threshold (i.e. taken as zero when in fact it is strictly positive), and when there are multiple censoring thresholds.

In the empirical application we find that, even though our results have the same expected sign as most of those found in the literature, the magnitude of coefficients are significantly lower. We believe that this result can have important policy implications, in the sense that studies might be overstating the degree to which monetary authorities react to fundamentals. In other words, the random component of policy might be larger than what was previously established with the use of other estimation methods. Consequently, we argue that a closer examination of flexible estimation methods are warranted, such as the one presented in this investigation, when drawing conclusions about the way central banks react to economic variables. Moreover, the importance of capturing systematic variation carries over to studies that aim to identify monetary policy shocks.[5]

This paper proceeds as follows: section 2 describes the methodology of the CLAD model. Sections 3 and 4 present simulations applied to a time-series setting, and some empirical results for the cases of Turkey and Colombia. Finally, section 5 concludes.

2 | CLAD METHODOLOGY

The CLAD model for a censoring threshold of zero was first proposed by Powell (1984) who considered a model based on the following form:

$$y_i = \max\{0, x_i'\beta_0 + \epsilon_i\},\ i = 1,\ldots,T \tag{1}$$

where x_i is a $k \times 1$ observed regressor vector and ϵ_i is a continuously distributed unobserved error term with a positive density function f_ε at zero and quantile function $\mathrm{Quant}_{0.5}(\epsilon_i|x_i) = 0$.

Under some regularity conditions (see Powell, 1984), it can be shown that a consistent estimator of β_0 is obtained as a solution to:

$$\min_\beta \frac{1}{T}\sum_i \left|y_i - \max\{0, x_i'\beta\}\right| \tag{2}$$

in which the first order condition is given by:

$$\frac{1}{2T}\sum_i I\left(x_i'\hat{\beta} > 0\right)\mathrm{sgn}\left\{y_i - x_i'\hat{\beta}\right\}x_i \tag{3}$$

where $I(\cdot)$ denotes the indicator function and $\mathrm{sgn}(\cdot)$ is the sign function. Under similar conditions Powell (1986) shows that the CLAD estimator is asymptotically normal: $\sqrt{T}\left(\hat{\beta} - \beta\right) \overset{\text{Dist.}}{\Rightarrow} N(0,\Sigma)$, where the variance matrix is exemplified by:

$$\Sigma = \frac{1}{4}\left(E[f_\varepsilon(0|x)I\left(x_i'\beta > 0\right)x'x]\right)^{-1} E\left[I\left(x_i'\beta > 0\right)x'x\right] \times \left(E[f_\varepsilon(0|x)I\left(x_i'\beta > 0\right)x'x]\right)^{-1} \tag{4}$$

where x stacks x_i' vertically and $f_\varepsilon(\cdot)$ denotes the density function of the error term.

An extension to a time series setting of CLAD is given by de Jong and Herrera (2008) who show that the LAD estimator is consistent even when the regressor vector x_i includes p-lags of the observed censored variable. The inclusion of the censored variable lags are used to model possible autocorrelation in the error terms of the model in Equation (1). The authors show that a sufficient condition for stationarity in the dynamic censored regression model is for the roots of the lag polynomial $\rho_{max}(z) = 1 - \sum_{i=1}^{p}\max(0,\rho_i)z^i$ to lie outside the unit circle, where the ρ_i's denote the coefficients of the lagged-dependent variables.[6]

In the related literature, several algorithms for the estimation of CLAD have been suggested and a useful discussion is found in Fitzenberger (1997). In this study we use the Iterative Linear Programming Algorithm (ILPA), as suggested by Buchinsky (1994).[7] The idea of the ILPA is to solve for $\hat{\beta}^{(j)}$ in the j^{th} iteration by using observations for which $x_{i'}\hat{\beta}^{(j-1)} > 0$ and to stop whenever the set of observations in two consecutive iterations are the same. In order to avoid a lack of robustness attributed to the starting value of the optimization process, we follow a genetic algorithm as described in Lucasius and Kateman (1993) to obtain the initial values used in the ILPA.

Finally, several methods for estimating the asymptotic covariance matrix, Σ, have been proposed for cases in which the distribution of the error term is independent of the regressors, that is, $f_{\varepsilon}(0|x) = f_{\varepsilon}(0)$. This, however, excludes (bounded) heteroskedastic behaviour in the error term, which we believe is essential in the context of high frequency data such as the case of foreign exchange intervention. For this reason we adopt a design matrix bootstrap, presented in Appendix A.[8]

3 | SIMULATIONS

This section analyses the performance of the CLAD estimator in a time series-setting through simulation exercises. For each exercise we consider random samples of different sizes (500 or 1,000). The estimation problem is hence a censored regression with three regressors and an intercept. We report bias, standard deviation and RMSE using both CLAD and Tobit estimators.

The data generating process is described as:

$$y_t^* = \beta_0 + \beta_1 y_{t-1}^* + \beta_2 x_{1,t-1} + \beta_3 x_{2,t-1} + \varepsilon_t \tag{5}$$

where $\beta_0 = 1$, $\beta_1 = 0.4$, $\beta_2 = 0.5$, and $\beta_3 = 1$.[9] Note that we only include one lag of the dependent variable, as in Romer and Romer (2004) and most of the related literature.[10] The initial values of the estimated coefficients are taken from a uniform distribution. The first regressor (x_{1t}) follows an AR(2) process, while the second (x_{2t}) follows an ARMA(1,1) process, both having unit mean and variance.[11] The correlation between both regressors is 0.4. Finally, we allow for autocorrelation in the error term that is modelled through the inclusion of the latent variable lag. The error term is taken from a standard normal distribution unless otherwise noted.

3.1 | Censoring threshold

For the first simulation exercise we consider a 75% degree of censoring, and a censoring threshold given by $\text{Quant}_{0.75}(y^*)$.[12] Following the results of Carson and Sun (2007) and Zuehlke (2010), we consider two cases: (i) a case in which we rescale the dependent variable to account for the non-zero threshold: $y_t^{\text{corr}} = y_t - \min_{\{t\}}\{y_t\}$, which we refer to as Case I[13] and (ii) a case in which we wrongly (and intentionally) assume that the threshold is zero, without rescaling, which we refer to as Case II.

The recording of a censoring value at zero rather than considering the actual censoring threshold is common in economic data mainly because of administrative recording practices (see Carson & Sun, 2007).[14]

Results presented in Table 1 suggest that the bias in the estimation (due to the incorrect specification of the censoring threshold) is severe for some of the coefficients. Namely, the bias in the estimation of β_2 and β_3 is over 0.5 for the CLAD method and over 1.0 for the Tobit model. The standard deviation of all coefficients is much larger. Nonetheless, the CLAD method seems to be more robust to the misspecification of the censoring threshold. We note that estimation results for the β_1 coefficient do not seem to change much when the dependent variable is rescaled.

TABLE 1 Simulation results for censoring threshold correction

	CLAD				Tobit			
	β_0	β_1	β_2	β_3	β_0	β_1	β_2	β_3
Case I								
$T = 500$								
Bias	−0.234	0.133	−0.074	−0.003	−0.548	0.185	−0.055	0.086
Sd	0.492	0.108	0.125	0.187	0.211	0.086	0.074	0.095
RMSE	0.545	0.171	0.145	0.187	0.588	0.204	0.092	0.128
$T = 1{,}000$								
Bias	−0.218	0.135	−0.070	−0.010	−0.528	0.184	−0.059	0.082
Sd	0.340	0.072	0.092	0.122	0.154	0.055	0.053	0.072
RMSE	0.404	0.152	0.115	0.122	0.550	0.192	0.079	0.109
Case II								
$T = 500$								
Bias	−4.745	0.210	0.862	2.118	−10.049	0.319	1.387	3.522
Sd	1.173	0.118	0.372	0.501	0.946	0.100	0.355	0.542
RMSE	4.888	0.240	0.939	2.177	10.093	0.334	1.432	3.563
$T = 1{,}000$								
Bias	−4.754	0.206	0.894	2.103	−10.037	0.313	1.365	3.532
Sd	0.775	0.078	0.277	0.363	0.739	0.072	0.261	0.345
RMSE	4.816	0.220	0.936	2.134	10.064	0.321	1.390	3.549

Authors' calculations. The table shows the CLAD estimation results when the censoring threshold is scaled (Case I) to account for the non-zero threshold and when it is not scaled and wrongly assumed to be zero (Case II). RMSE denotes the root mean squared error; Sd denotes the empirical standard deviation of the estimates.

3.2 | Degree of censoring

For the second simulation the censoring threshold is $\text{Quant}_\theta(y^*)$, $\theta \in \{0.25, 0.5, 0.75\}$. Before the estimation procedure, the dependent variable is rescaled to account for the non-zero censoring threshold.

The degree of censoring in the sample is crucial to assess the efficiency of the proposed estimation method. In fact, Fitzenberger (1997) shows that all practical algorithms for the CLAD estimation perform poorly when there is a high percentage of censoring, which generally applies to foreign exchange intervention data.

Table 2 shows the estimation results for different degrees of censoring. The resulting bias for the β_2 and β_3 coefficients do not change much as the censoring percentage grows, but the growth in the standard deviation is much steeper. Results for β_1 suggest that, as the censoring percentage grows, the bias increases. This can be explained by noting that Equation (5) depends on the lagged latent variable. Hence, the difference between the lagged latent and observed variable grows as the censoring in the sample increases. The RMSE diminishes as the sample size increases for all censoring percentages. The CLAD estimation method has a lower RMSE for the β_0 and β_1 coefficients.

3.3 | Heteroskedastic behaviour

For the third simulation exercise we consider different distributions for the conditional error terms $\varepsilon_t|\psi_{t-1}$ (Gaussian, t with 5 degrees of freedom and Laplace).[15] The error term follows a GARCH(1,1)

TABLE 2 Simulation results for different censoring percentages

	CLAD				Tobit			
	β_0	β_1	β_2	β_3	β_0	β_1	β_2	β_3
25%								
$T = 500$								
Bias	−0.136	0.042	−0.020	0.012	−0.179	0.048	−0.021	0.028
Sd	0.168	0.043	0.070	0.078	0.122	0.035	0.052	0.057
RMSE	0.216	0.060	0.072	0.079	0.216	0.059	0.056	0.063
$T = 1{,}000$								
Bias	−0.133	0.044	−0.016	0.009	−0.175	0.048	−0.015	0.023
Sd	0.112	0.030	0.047	0.059	0.080	0.024	0.036	0.042
RMSE	0.174	0.053	0.050	0.060	0.193	0.054	0.039	0.048
50%								
$T = 500$								
Bias	−0.200	0.079	−0.045	0.024	−0.327	0.103	−0.041	0.054
Sd	0.242	0.060	0.089	0.108	0.145	0.047	0.060	0.070
RMSE	0.314	0.099	0.099	0.111	0.358	0.113	0.073	0.089
$T = 1{,}000$								
Bias	−0.192	0.084	−0.034	0.009	−0.323	0.102	−0.035	0.049
Sd	0.162	0.043	0.062	0.078	0.093	0.034	0.042	0.051
RMSE	0.251	0.094	0.071	0.078	0.337	0.107	0.055	0.071
75%								
$T = 500$								
Bias	−0.245	0.125	−0.081	0.007	−0.530	0.183	−0.069	0.087
Sd	0.500	0.109	0.132	0.175	0.224	0.079	0.078	0.096
RMSE	0.557	0.166	0.155	0.175	0.575	0.199	0.104	0.130
$T = 1{,}000$								
Bias	−0.239	0.131	−0.072	−0.001	−0.527	0.179	−0.064	0.082
Sd	0.368	0.077	0.092	0.131	0.150	0.058	0.053	0.064
RMSE	0.439	0.153	0.117	0.131	0.548	0.188	0.083	0.104

Authors' calculations. The table shows the CLAD estimation results when the censoring percentage in the sample changes. RMSE denotes the root mean squared error; Sd denotes the empirical standard deviation of the estimates.

process, where $\sigma_t^2 = \gamma + 0.3\epsilon_{t-1}^2 + 0.65\epsilon\sigma_{t-1}^2$ and γ is a number adapted to make the unconditional variance of the process equal unity. The censoring threshold is given by $\text{Quant}_{0.75}(y^*)$. As before, the dependent variable is rescaled to account for the non-zero censoring threshold.

Table 3 shows the estimation results for the case of conditional heteroskedasticity. Results suggest that for all conditional distributions considered, the CLAD estimator outperforms the Tobit in terms of bias. The RMSE of all coefficients is lower using the CLAD method except for β_2. This last result is expected, since, as suggested in Arabmazar and Schmidt (1981), when heteroskedasticity is present, the Tobit model (which assumes homoskedasticity) is inconsistent.

TABLE 3 Simulation results for conditional heteroskedasticity—GARCH(1,1)

	CLAD				Tobit			
	β_0	β_1	β_2	β_3	β_0	β_1	β_2	β_3
N								
$T = 500$								
Bias	−0.224	0.122	−0.070	0.013	−0.599	0.215	−0.044	0.098
Sd	0.402	0.100	0.103	0.139	0.336	0.107	0.076	0.119
RMSE	0.461	0.158	0.125	0.140	0.687	0.240	0.088	0.154
$T = 1,000$								
Bias	−0.220	0.122	−0.071	0.008	−0.626	0.215	−0.044	0.099
Sd	0.262	0.064	0.072	0.102	0.280	0.077	0.058	0.088
RMSE	0.342	0.137	0.101	0.102	0.686	0.229	0.073	0.132
T5								
$T = 500$								
Bias	−0.196	0.116	−0.068	0.018	−0.591	0.196	−0.037	0.099
Sd	0.336	0.077	0.076	0.120	0.377	0.087	0.071	0.123
RMSE	0.389	0.139	0.102	0.121	0.700	0.215	0.080	0.158
$T = 1,000$								
Bias	−0.212	0.116	−0.067	0.017	−0.661	0.203	−0.037	0.116
Sd	0.208	0.059	0.054	0.077	0.340	0.079	0.062	0.104
RMSE	0.297	0.130	0.086	0.079	0.743	0.218	0.073	0.156
Lap								
$T = 500$								
Bias	−0.190	0.109	−0.067	0.012	−0.617	0.201	−0.036	0.100
Sd	0.267	0.068	0.070	0.098	0.377	0.093	0.080	0.109
RMSE	0.328	0.129	0.097	0.098	0.723	0.221	0.088	0.148
$T = 1,000$								
Bias	−0.223	0.106	−0.059	0.021	−0.691	0.205	−0.029	0.116
Sd	0.190	0.053	0.049	0.069	0.320	0.076	0.057	0.100
RMSE	0.293	0.119	0.077	0.072	0.762	0.219	0.064	0.153

Authors' calculations. The table shows the CLAD estimation results when the error term follows a GARCH(1,1) process. RMSE denotes the root mean squared error; Sd denotes the empirical standard deviation of the estimates.

3.4 | Error distribution

For the fourth simulation exercise we consider different distribution functions for the error term, including: Gaussian, *t*-student with 5 degrees of freedom, and Laplace. The latter two distributions capture the heavy tailed behaviour of high frequency financial data. The censoring threshold is given by $\text{Quant}_{0.75}(y^*)$. Also, the dependent variable is rescaled to account for the non-zero censoring threshold.

As can be seen in Table 4, the change in the distribution of the error term leads to results that are very similar to those of Table 3 (regarding GARCH processes). The CLAD estimator seems more robust to non-normality in the error term than the Tobit model, which confirms the result found in Arabmazar and Schmidt (1982).

TABLE 4 Simulation results for different error distributions

	CLAD				Tobit			
	β_0	β_1	β_2	β_3	β_0	β_1	β_2	β_3
N								
$T = 500$								
Bias	−0.165	0.122	−0.083	−0.014	−0.516	0.191	−0.055	0.083
Sd	0.471	0.106	0.125	0.186	0.244	0.079	0.082	0.105
RMSE	0.499	0.161	0.150	0.187	0.571	0.206	0.099	0.134
$T = 1{,}000$								
Bias	−0.257	0.132	−0.080	0.008	−0.539	0.179	−0.062	0.089
Sd	0.376	0.075	0.090	0.129	0.158	0.059	0.054	0.066
RMSE	0.455	0.152	0.120	0.129	0.562	0.189	0.082	0.111
T5								
$T = 500$								
Bias	−0.248	0.126	−0.077	0.013	−0.637	0.191	−0.055	0.123
Sd	0.418	0.086	0.111	0.157	0.275	0.075	0.076	0.113
RMSE	0.486	0.153	0.135	0.157	0.694	0.205	0.094	0.167
$T = 1{,}000$								
Bias	−0.249	0.119	−0.074	0.006	−0.662	0.189	−0.050	0.118
Sd	0.305	0.065	0.083	0.112	0.191	0.059	0.057	0.079
RMSE	0.393	0.136	0.111	0.112	0.689	0.198	0.076	0.141
Lap								
$T = 500$								
Bias	−0.252	0.119	−0.070	0.001	−0.650	0.193	−0.056	0.111
Sd	0.375	0.079	0.094	0.135	0.242	0.083	0.074	0.108
RMSE	0.452	0.142	0.117	0.135	0.694	0.210	0.093	0.155
$T = 1{,}000$								
Bias	−0.231	0.115	−0.058	0.004	−0.656	0.197	−0.045	0.108
Sd	0.270	0.051	0.070	0.093	0.185	0.054	0.057	0.076
RMSE	0.355	0.126	0.091	0.093	0.682	0.204	0.073	0.131

Authors' calculations. The table shows the CLAD estimation results when the error term has different distributions. RMSE denotes the root mean squared error; Sd denotes the empirical standard deviation of the estimates.

3.5 | Multiple thresholds

In this simulation exercise, we consider multiple thresholds. In particular, we consider 10 sub-samples of equal length, each with a different threshold, according to two different rules. The type I censoring threshold is given by $\text{Quant}_{0.75}\left(y_i^*\right)$, where y_i denotes the i^{th} sub-sample, while the type II censoring threshold is given by $\text{Quant}_{0.75}\left(y_i^*\right)$, where $\alpha = 0.25$ if i is even and $\alpha = 0.75$ otherwise. Hence, the dependent variable is wrongly (and intentionally) rescaled by considering the minimum value of the whole sample, as described in the previous sections.[16]

In a foreign exchange intervention setting, the assumption of a unique censoring threshold across the entire sample is somewhat questionable given that the board of directors of a central bank is subject to the election of new members that can in turn react to different fundamentals. This brings to mind the possibility of several censoring thresholds during the time period considered.

Table 5 shows the estimation results for two different types of censoring. As shown, the bias appears in estimations when the difference between the censoring thresholds of the sub-samples and the value used to rescale the dependent variable is larger. This occurs in the type-II censoring threshold, mainly because the difference between the thresholds in the sub-sample is larger than in the type-I censoring. Once again, we find that the CLAD estimator seems to outperform the Tobit estimator both in terms of bias and RMSE for almost all coefficients.

3.6 | Summary of simulation results

The simulation exercises presented in this investigation suggest that the CLAD estimator used in a time-series framework outperforms the Tobit model in terms of bias and RMSE when there is conditional heteroskedasticity (GARCH) or when the normality assumption in the error term does not hold. This result is in line with Arabmazar and Schmidt (1981, 1982). Similarly, we find that the CLAD estimator is more robust than the Tobit model when there is misspecification of the censoring threshold (i.e. taken as zero when in fact it is strictly positive), and when there are multiple censoring thresholds while only one is taken into account.

TABLE 5 Simulation results for multiple thresholds

	CLAD				Tobit			
	β_0	β_1	β_2	β_3	β_0	β_1	β_2	β_3
Type I								
$T = 500$								
Bias	−0.492	0.138	0.000	0.159	−1.157	0.207	0.039	0.325
Sd	0.465	0.092	0.146	0.185	0.382	0.079	0.108	0.153
RMSE	0.677	0.166	0.146	0.244	1.218	0.221	0.115	0.360
$T = 1{,}000$								
Bias	−0.577	0.143	0.036	0.183	−1.389	0.209	0.084	0.394
Sd	0.354	0.071	0.108	0.136	0.337	0.063	0.088	0.121
RMSE	0.676	0.160	0.113	0.228	1.429	0.219	0.122	0.412
Type II								
$T = 500$								
Bias	−0.524	0.189	0.141	0.278	−1.192	0.208	0.187	0.414
Sd	0.332	0.068	0.142	0.170	0.238	0.060	0.110	0.131
RMSE	0.620	0.201	0.200	0.326	1.216	0.217	0.217	0.435
$T = 1{,}000$								
Bias	−0.286	0.189	0.131	0.189	−0.977	0.198	0.195	0.356
Sd	0.197	0.050	0.087	0.107	0.179	0.043	0.085	0.100
RMSE	0.347	0.195	0.158	0.217	0.993	0.203	0.213	0.370

Authors' calculations. The table shows the CLAD estimation results when there are multiple thresholds. The Type I censoring threshold is given by $\mathrm{Quant}_{0.75}(y_i^*)$, where y_i denotes the i-th sub-sample, while the Type II censoring threshold is given by $\mathrm{Quant}_{\alpha}(y_i^*)$, where $\alpha = 0.25$ if i is even and $\alpha = 0.75$ otherwise. RMSE denotes the root mean squared error; Sd denotes the empirical standard deviation of the estimates.

As expected, increasing the degree of censoring leads to bias in the lagged-coefficient term along with an increase in the standard deviation of all the estimated coefficients.

4 | EMPIRICAL ESTIMATION

4.1 | Data

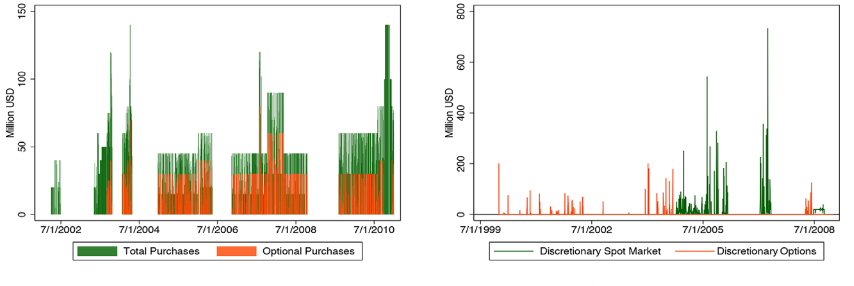
Our data covers the period January 2002 through May 2010 for the Turkish case and December 1999 through October 2008 for the Colombian case. Prior to these dates, exchange rate bands and a fixed exchange rate regime were enacted for Colombia and Turkey, respectively. Also, following 2010, both countries adopted additional monetary instruments: a reserve option mechanism and an interest rate corridor in Turkey, and constant (daily) foreign exchange interventions in Colombia. Our selected sample thus poses a methodological advantage given that we avoid making further assumptions to model individual policies.

For the Colombian case, we use purchases of USD conducted in the spot market (22.8 billion) as well as discretionary interventions through foreign exchange rate options (3.3 billion). Alternatively, for the Turkish case, we use optional purchases (20.4 billion) which consisted of a discretionary amount of trading that took place during the day of an announced auction.[17] These foreign exchange transactions, denoted as FXI henceforth, are depicted in Figure 1.

A crucial step needed to model foreign exchange intervention is to choose an informative history of economic variables. In other words, to capture the relevant information that monetary authorities used when setting their policy decisions (see Villamizar-Villegas, 2016). In the related literature, surveys such as Dornbusch (1980), Meese and Rogoff (1983), Dominguez and Frankel (1993), Edison (1993), Sarno and Taylor (2001), Neely (2005), and Menkhoff (2013), describe the inclusion of some measure of exchange rate misalignment (deviations from a targeted level or moving average). Some studies also include some measure of currency risk premium. Variables used often range from the estimated coefficient of risk aversion (Disyatat & Galati, 2007; Frankel & Dominguez, 1993) to some measure of exchange rate volatility (Baillie & Osterberg, 1997; Onder & Villamizar-Villegas, 2015; Villamizar-Villegas, 2016).

Finally, some works follow the goods and services market approach described in Mussa (1976) in order to model exchange rate behaviour when the purchasing power parity (PPP) condition holds. Studies that follow this approach generally include macroeconomic variables such as inflation and output (Gartner, 1987; Mastropasqua, Micossi, & Rinaldi, 1989; Tullio & Ronci, 1996).

As such, our explanatory variables include exchange rate changes, a measure of exchange rate misalignment (ERM), exchange rate volatility (Vol), macroeconomic indicators (e.g. inflation and



(a) Turkey (b) Colombia

FIGURE 1 Foreign exchange interventions for Colombia and Turkey 2000–2010 output) and other variables such as episodes of capital controls that could have potentially influenced official intervention.[18] We also considered yearly dummy variables and report only those with statistical significance. All variables are daily, except for inflation and output which had a monthly frequency. A more in-depth description of each variable is found in Appendix B. Also, unit-root tests are reported in Appendix C, Table C1.

4.2 | Estimating foreign exchange policy functions

Tables 6 and 7 present results using the proposed CLAD methodology and compare them with the benchmark Tobit model. For the case of Colombia, the central bank tried to depreciate domestic currency (by purchasing USD) whenever interventions were conducted the day before, when the exchange rate appreciated (relative to its forecasted equilibrium value), whenever the central bank was a net debtor with respect to the financial sector, and to a lesser extent, when capital controls were enacted (see Table 6). Similarly, for the case of Turkey, the central bank tried to depreciate domestic currency whenever interventions were conducted the day before, when inflation was low (with respect to the yearly target), and whenever output increased (see Table 7).

TABLE 6 Estimation results for Colombia

	CLAD	CLAD	Tobit
Censoring threshold:	**Fixed**	**Dynamic**	**Fixed**
FXI_{t-1}	0.20***	0.21***	0.59***
	(0.07)	(0.08)	(0.06)
ERM_t	−0.71*	−0.70	−5.04***
	(0.39)	(0.53)	(0.86)
Vol_t	9.55	10.70	5.76***
	(7.72)	(6.79)	(1.53)
D_{Tax}	−32.19***	−47.96***	−145.2***
	(13.41)	(16.68)	(18.10)
D_{Net}	13.87**	13.60**	0.85
	(6.69)	(5.94)	(9.53)
Intercept	−32.55***	−38.33***	−90.50***
	(11.02)	(12.81)	(8.58)
D_{2004}	33.90**	38.67***	80.00***
	(14.72)	(14.81)	(9.74)
D_{2005}	36.06***	42.07***	100.04***
	(12.21)	(14.15)	(8.84)
D_{2006}	34.07***	41.16***	19.89*
	(13.70)	(14.70)	(10.42)
D_{2007}	73.33***	95.61***	33.72***
	(24.28)	(24.68)	(8.99)

Estimation results for Colombia considering: (i) a fixed censoring threshold; (ii) a dynamic censoring threshold; and (iii) a standard Tobit model. FXI corresponds to foreign exchange interventions, ERM to exchange rate misalignments, Vol_t to exchange rate volatility, D_{Net} to the net credit/debit position of the central bank, and D_{Tax} to a period of capital controls. Standard errors are reported in parentheses. *,**, *** indicate significance at the 10%, 5%, and 1% level, respectively.

TABLE 7 Estimation results for Turkey

	CLAD	Tobit
Censoring threshold:	**Fixed**	**Fixed**
FXI_{t-1}	0.540***	0.826***
	(0.117)	(0.047)
$\pi_t - \pi^*$	−0.183***	−0.125***
	(0.000)	(0.021)
ΔY_t	$1.43^{***} \times 10^{-4}$	$6.53^{***} \times 10^{-4}$
	(4.15×10^{-5})	(1.03×10^{-4})
ERM_t	1.01×10^{-9}	$9.19^{***} \times 10^{-4}$
	(4.18×10^{-9})	(1.92×10^{-4})
Intercept	−1.045***	−0.017***
	(0.002)	(0.002)
D_{2004}	0.549***	−0.15***
	(0.002)	(0.002)
D_{2008}	2.163***	0.018***
	(0.019)	(0.002)
D_{2009}	0.863***	−0.003
	(0.011)	(0.003)
D_{2010}	1.038***	0.007**
	(0.008)	(0.003)

Estimation results for Turkey for the fixed censoring threshold case. FXI corresponds to foreign exchange interventions, $\pi_t - \pi^*$ to inflation minus yearly target, ERM to exchange rate misalignments, and ΔY to industrial output growth. Standard errors are reported in parentheses. *,**,*** indicate significance at the 10%, 5%, and 1% level, respectively.

In general, these results suggest that foreign exchange interventions are an important policy tool for central banks. In fact, both countries target exchange rate behaviour, and in the case of imperfect sterilization (the case of Turkey), we attribute the negative (positive) effect of inflation (output) as 'leaning with the wind' policies. Given the importance of foreign exchange policies, we believe that additional studies are warranted in order to address the effects of multiple policies. As stated in Ordoñez-Callamand, Hernandez-Leal, and Villamizar-Villegas (2017), 'policy instruments can be inadvertently collinear, leading to monetary indeterminacy, and identification failures' (see also Kim, 2003). Ultimately, in the context of the *monetary trilemma*, it raises the question of whether central banks sometimes overreach when simultaneously allowing for: (i) free capital flows; (ii) autonomous monetary policy; and (iii) a managed exchange rate.

While these results have the same expected sign as the ones found in Herrera and Ozbay (2005), Kamil (2008), Echavarría et al. (2017), Onder and Villamizar-Villegas (2015), and Villamizar-Villegas (2016), the magnitude of the coefficients are, in absolute terms, significantly lower. This can also be corroborated by comparing the CLAD and Tobit coefficients presented in columns 4 and 3 of Tables 6 and 7, respectively (see in particular the coefficients of the lagged dependent variable).[19] We believe that this result can have important policy implications in the sense that studies might be overstating the degree to which monetary authorities react to fundamentals. Also, we find a significantly lower coefficient of lagged intervention. Namely, while most studies find a value of the coefficient close to unity, we find values of 0.2 and 0.5 for Colombia and Turkey, respectively. This shows a lower persistence when conducting foreign exchange interventions.

Finally, we analysed whether changes in the composition of the board of directors within the Central Bank of Colombia had an effect on the estimated threshold.[20] In essence, we inquired whether Colombia exhibited multiple thresholds that, if ignored, would yield inconsistent estimates (see section 3). Given the similarities between columns 2 and 3 reported in Table 6, we conclude that changes in the board did not alter the censoring threshold in the foreign exchange policy function.

5 | CONCLUSION

In this paper we take a fresh look at a key issue in the international finance and central bank literature, namely to allow for a more flexible estimation of policy. Essentially, when modelling foreign exchange intervention, authors may choose different variables to model policy, adopt different functional forms, and treat contextual characteristics in various ways. A caveat, however, is that the use of heavily dependent parametric models sometimes conditions the validity of results.

We hence propose a method that is robust to several model specifications, including heteroskedasticity and different distributions in the error term. Specifically, we estimate a Censored Least Absolute Deviation model (CLAD) applied to a time-series framework. We centre our investigation on policies that relate to foreign exchange intervention, where central banks have generally conducted numerous and large-scale purchases of foreign currency but a general absence of sales. In fact, this asymmetry has been coined as a 'fear of floating' and has motivated the use of censored parametric models, such as the Tobit model.

In simulation exercises we find that our proposed method (CLAD) outperforms the Tobit model on several fronts (bias and root-mean-square-error), thus enabling more precise estimates. Additionally, we empirically estimate foreign exchange policies for two emerging market economies: the cases of Turkey and Colombia during the period 2000–2010. We find marked differences with the related literature, especially regarding the magnitude of the coefficients (ours are significantly lower). This result can have important policy implications in the sense that studies might be overstating the degree to which monetary authorities react to fundamentals. We argue that a closer examination of flexible estimation methods is warranted when drawing conclusions about the way central banks react to economic variables. Moreover, the importance of capturing systematic variation carries over to studies that aim to correctly identify monetary policy shocks.

ENDNOTES

[1] A compilation of empirical findings can be found in Meese and Rogoff (1988), Dominguez and Frankel (1993), Edison (1993), Dominguez (2003), Neely (2005), Menkhoff (2010), and Villamizar-Villegas and Perez-Reyna (2017).

[2] See Almekinders and Eijffinger (1996), Guimaraes and Karacadag (2005), Huang (2007), Jun (2008), Humala and Rodríguez (2010), Rincón and Toro (2010), and Echavarría et al. (2017).

[3] See Kim, Kortian, and Sheen (2000), Kamil (2008), Adler and Tovar (2014), Villamizar-Villegas (2016), and Onder and Villamizar-Villegas (2015).

[4] Studies that follow a similar approach include Kamil (2008), Villamizar-Villegas (2016), and Onder and Villamizar-Villegas (2015).

[5] This is particularly useful for studies that analyse multiple policies, such as Kim (2003, 2005) and Bjørnland (2008, 2009).

[6] One should note that once a sufficient number of dependent variable lags are included, the resulting error term in Equation (1) should be white noise, so Equation (4) would be asymptotically correct.

[7] Fitzenberger (1997) shows that ILPA is not able to interpolate censored observations. However, modifying this aspect does not lead to meaningful improvements on the estimator (see Fitzenberger, 1994).

[8] For cases in which heteroskedasticity is present, only kernel estimation and design matrix bootstrap are robust to the estimation of the asymptotic covariance matrix (see Buchinsky, 1995).

[9] Due to the way that we impose the 75% censoring, the coefficient of β_0 is a random variable with mean $1 - 0.6*\mathbf{E}[\text{Quant}_{0.75}(y^*)]$.

[10] Romer and Romer (2004) argue, for example, that the inclusion of the lagged policy rate, in levels, captures tendencies towards mean reversion in the Federal Reserve's behaviour.

[11] The coefficients are taken from a uniform distribution. For the first regressor, $\phi_1 = -0.3$ and $\phi_2 = 0.45$. For the second regressor, $\phi_1 = 0.5$ and $\theta_1 = 0.3$. Note that in both cases the regressors are stationary.

[12] The 75% threshold is similar to that of our data for Colombia and Turkey, and is in accord with the high degree of censoring of most of the foreign exchange intervention literature.

[13] In empirical applications where the censoring threshold differs from zero, the threshold can be taken as the minimum value of y_t as suggested in Carson and Sun (2007) and Zuehlke (2010).

[14] Moreover, some of the statistical software available defaults to a zero censoring threshold when estimating this type of regression model, so misspecification of the censoring threshold is liable to occur in empirical applications.

[15] We use ψ_{t-1} to denote the information history up to $t-1$.

[16] Note that these simulation exercises are misspecified since the CLAD estimations consider only one threshold.

[17] We exclude unannounced purchases and sales for the Turkish case, due to the few observations available.

[18] Variables included for each country were based on data availability and contextual characteristics. For example, capital control episodes were included only for the Colombian case given that Turkey only enacted controls in the later part of 2010, which falls outside our sample period (see Magud, Reinhart, & Rogoff, 2011).

[19] One should note that in this case Tobit and CLAD estimators are comparable because under the normality assumption the mean and the median of the distribution of $(y_t|x_t)$ are the same.

[20] Due to lack of information, we omitted this exercise for the Turkish case.

[21] For a detailed use block bootstrap in a quantile regression setting, see Fitzenberger (1998).

[22] In practice, several replications should be made (we suggest at least 100).

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APPENDIX A: BOOTSTRAP ALGORITHM

We consider a stationary bootstrap algorithm in order to estimate the error-covariance matrix. To account for the (possibly) weak dependence of the data over time and to allow for heteroskedastic behaviour in the error term, we consider the use of the stationary bootstrap proposed by Politis and Romano (1994) over the traditional block bootstrap of Kunsch (1989).[21]

The model of interest can be written as

$$y_i = \max\{0, x_i'\beta_0 + \epsilon_i\}, \quad i = 1,\ldots,T \tag{6}$$

where x_i is a $k \times 1$ vector that contains the regressors. In our case x_i' contains the lag of the observed variable y_i, as well as other variables of interest. Let $z_i = (y_i, x_i')'$ define a subset of these T observations, which we call a block, as:

$$B_{i,b} := [z_i, z_{i+1}, \ldots, z_{i+b-1}]'$$

The algorithm of the block bootstrap implementation can be described as follows

1. Choose a constant $p \in (0,1)$. This parameter is related to the expected bootstrap block size (see Politis & Romano, 1994).
2. Sample $l_1, l_2, \ldots, l_k$ independent and identically distributed random variables from the geometric distribution with parameter p from the previous step, where k is a number such that $\sum_{j=1}^{k} l_j = T$.
3. Sample $u_1, u_2, \ldots, u_k$ independent and identically distributed random variables from the discrete uniform distribution in $\{1,\ldots,T\}$.
4. Sample a sequence of k blocks of random lengths $l_1, l_2, \ldots, l_k$ with observations associated with $u_1, u_2, \ldots, u_k$: $B_{u_1,l_1}, B_{u_2,l_2}, \ldots, B_{u_k,l_k}$. Note that the variable u_j is related to the point where block j begins while l_j denotes the size of that block. For example, if $u_j = 3$ and $l_j = 5$ then the *jth*-block, B_{u_j,l_j}, is defined as $[z_3, z_4, z_5, z_6, z_7]'$.
5. Generate a pseudo-series Z^*, where $Z^* := [B_{u_1,l_1}', B_{u_2,l_2}', , B_{u_k,l_k}']'$. This can be rewritten as

$$Z^* = \begin{bmatrix} z_1'^* \\ z_2'^* \\ \vdots \\ z_T'^* \end{bmatrix} = \begin{bmatrix} y_1^* & x_1'^* \\ y_2^* & x_2'^* \\ \vdots & \vdots \\ y_T^* & x_T'^* \end{bmatrix}$$

For example, if $T = 5$, $k = 2$, $l_1 = 3$, $l_2 = 2$, $u_1 = 2$, $u_2 = 4$, then

$$Z^* = \begin{bmatrix} B_{u_1,l_1} \\ \\ B_{u_2,l_2} \end{bmatrix} = \begin{bmatrix} z_1'^* \\ z_2'^* \\ z_3'^* \\ \\ z_4'^* \\ z_5'^* \end{bmatrix} = \begin{bmatrix} y_1^* & x_1'^* \\ y_2^* & x_2'^* \\ y_3^* & x_3'^* \\ \\ y_4^* & x_5'^* \\ y_5^* & x_5'^* \end{bmatrix} = \begin{bmatrix} z_2' \\ z_3' \\ z_4' \\ \\ z_4' \\ z_5' \end{bmatrix} = \begin{bmatrix} y_2 & x_2' \\ y_3 & x_3' \\ y_4 & x_4' \\ \\ y_4 & x_4' \\ y_5 & x_5' \end{bmatrix}'$$

Note that the series with asterisk denote resampled observations and the series without asterisk correspond to the original data.
6. Using the pseudo-series y_i^* and $x_i'^*$, reestimate the model in (6) using the CLAD estimator.

7. Repeat steps 2–6 for each bootstrap replication.[22]

The main advantage of the described procedure is that the pseudo-series generated is stationary conditional on the original series. To address end corrections, we follow the same procedure as in the circular bootstrap (Politis & Romano, 1991).

It is easy to see that for the algorithm described above, the expected length of the block is $1/p$. This feature makes the stationary bootstrap more robust to misspecification of the block size than the other types of block bootstrap. When bootstrapping, we construct the standard deviation using the robust scale estimator S_n as proposed by Rousseeuw and Croux (1993).

APPENDIX B: DATA DESCRIPTION

For the Colombian case we use the following variables as covariates:

- Lag of foreign exchange intervention (FXI_{t-1}), (daily).
- Exchange rate misalignment (ERM_t): Log-difference between the exchange rate and the average forecasted equilibrium value of seven in-house models used by the Central Bank of Colombia, (daily).
- Exchange rate volatility (Vol_t): Exchange rate returns with respect to its 20-day moving average, (daily).
- Capital controls (D_{Tax}): Period of capital controls implemented between May 2007 and October 2008, (dummy variable).
- Net credit/debit position (D_{Net}): Dummy variable switched on whenever the central bank was a net debtor with respect to the financial sector (i.e. excess liquidity), (dummy variable).
- Year dummies (D_{year}).

Similarly, variables used as covariates for the Turkish case include:

- Lag of foreign exchange intervention (FXI_{t-1}), (daily).
- Exchange rate misalignment (ERM_t): 20-day exchange rate change (log-difference), (daily).
- Inflation minus yearly target $\pi_t - \pi^*$, (monthly).
- Industrial output growth (ΔY_t), (monthly).
- Year dummies $\left(D_{\text{year}}\right)$.

APPENDIX C: UNIT ROOT TEST

TABLE C1 Elliott–Rothenberg-stock test for unit root

Variable (up to 28 lags)	*t*-statistic	1% critical value	10% critical value
Turkey			
FXI_t	−3.791	−3.480	−2.570
$\pi_t - \pi^*$	−2.640	−3.480	−2.570
ERM_t	−6.336	−3.480	−2.570
ΔY_t	−3.070	−3.480	−2.570
Colombia			
FXI_t	−5.517	−3.480	−2.570
Vol_t	−8.413	−3.480	−2.570
ERM_t	−2.812	−3.480	−2.570
D_{Net}	−6.131	−3.480	−2.570
D_{Tax}	−7.599	−3.480	−2.570

FXI corresponds to foreign exchange interventions, $\pi - \pi^*$ to inflation minus yearly target, ERM to exchange rate misalignments, ΔY to industrial output growth, Vol_t to exchange rate volatility, D_{Net} to the net credit/debit position of the central bank, and D_{Tax} to a period of capital controls. The minimum lag is determined using the modified Akaike's information criterion (MAIC). All variables reject the null hypothesis of a unit root at the 10% level, and most at the 1% level.