

# Declining Search Frictions, Unemployment, and Growth Revisited\*

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## Abstract

This paper revisits the conditions under which search models generate balanced growth paths (BGPs)—equilibria where unemployment, vacancies, and job flows remain stable even as search frictions decline. Martellini and Menzio (2020) claim that such paths exist only when matches are “inspection goods” and match quality follows a Pareto distribution. We show that these conditions are sufficient but not necessary. Their implementation imposes a multiplicative scaling of match quality, forcing the reservation quality to grow at a constant, strictly positive rate. This assumption mechanically ties declining frictions to long-term growth and yields counterfactual implications of eliminating search frictions—persistent unemployment and infinite welfare gains. Relaxing this restriction, balanced growth can arise under alternative forms of scaling, such as additive transformations that restore stationarity without Pareto tails or inspection. We further show that biased technological progress, when vacancies and unemployed workers are complementary inputs, also generates well-behaved BGPs with finite welfare gains and vanishing unemployment as search frictions disappear.

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”If search frictions in the labor market have diminished over the last 90 years, why do we not see a secular inward shift of the Beveridge curve, a secular negative trend in the unemployment rate, and a secular rise in the UE rate?... We seek a balanced growth path (BGP) for this economy, that is, an equilibrium along which unemployment, vacancies, UE, and EU rates are constant over time... A BGP exist iff (a) the quality of a firm-worker match is a sample from a Pareto distribution with some tail coefficient... and (b) the workers’ benefit from unemployment and the firms’ cost of maintaining a vacancy grow at the same rate as average productivity. The assumption that matches are inspection goods could be considered the third condition for the existence of a BGP.” (Martellini and Menzio, 2020, pp. 4392).

## 1 Introduction

Despite dramatic improvements in job-search technology over the past century—from newspaper classifieds to online platforms to algorithmic matching—aggregate labor market outcomes have remained remarkably stable. Unemployment rates show no secular decline, the Beveridge curve has not shifted inward over the long run, and job-finding and separation rates appear stationary. This apparent disconnect between technological progress and stable labor-market outcomes poses a fundamental puzzle for macroeconomic theory: how can declining search frictions coexist with stationary employment dynamics?

Martellini and Menzio (2020) (hereafter MM) offer an influential answer. In the spirit of King, Plosser, and Rebelo (1988), they seek necessary and sufficient conditions for balanced growth in search-theoretic models of unemployment. Their solution is strikingly sharp: a balanced growth path exists if and only if firm-worker matches are “inspection goods” and match quality follows a Pareto distribution.<sup>1</sup> Under these assumptions, the long tail of the Pareto distribution induces increasingly selective matching that offsets the effects of declining frictions, thereby preserving stationarity while generating long-run growth.

This paper revisits and challenges that characterization. We show that MM’s conditions are *sufficient but not necessary*. The key lies in how they operationalize the notion of balanced growth. By definition, a BGP refers to observable aggregates—unemployment, vacancies, and job flows—that remain constant over time. Ensuring this stationarity, however, requires the underlying distribution of match qualities—an unobservable variable—to evolve in a stationary way once properly detrended. MM achieve this by imposing a specific form of detrending: *multiplicative scaling* of match quality, which forces the reservation quality to grow at a constant and strictly positive rate. This choice enforces proportional (scale-invariant) adjustment in match quality and mechanically links declining

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<sup>1</sup>Inspection means that when a worker and a vacancy meet, the match-specific productivity is revealed and the parties decide whether to form an employment relationship.

search frictions to long-term growth. Yet multiplicative detrending is not implied by the BGP definition itself. Once detrending is treated more generally, the Pareto distribution requirement is no longer necessary for balanced growth.

We demonstrate that alternative time transformations can restore stationarity without invoking Pareto tails. Under *exponential sampling* of match quality, improvements in search efficiency cause the reservation quality to drift *linearly* over time, rather than scale proportionally as in the Pareto case. This additive transformation of time ensures that the distribution of accepted matches remains stationary even as search efficiency improves exponentially. In equilibrium, self-selection adjusts just enough to offset the destabilizing effect of improved matching—restoring stationarity but without generating long-run growth.

The restrictive structure implied by multiplicative scaling also generates counterfactual implications. In the MM setup, unemployment persists even when posting vacancies is costless, and the welfare gains from eliminating search frictions are infinite. In contrast, the canonical Diamond–Mortensen–Pissarides (DMP) framework predicts that unemployment vanishes as posting costs fall and that welfare gains are finite, bounded by the forgone output of unemployed workers. Under exponential sampling, one core DMP prediction is restored—unemployment vanishes as posting costs fall—but, as in the Pareto inspection case, the welfare gains from eliminating frictions remain unbounded because ever-easier matching induces ever-higher reservation quality when match quality is unbounded above.

Balanced growth can also arise outside the inspection framework. When technological progress is input-biased and vacancies and unemployed workers are complementary inputs in the matching function, the economy converges to a well-behaved BGP with stationary unemployment, tightness, and job-finding rates. In this setting, unemployment vanishes as frictions disappear only if technological progress is worker-augmenting. Unlike inspection models, the welfare gains from eliminating frictions are finite. However, the resulting BGP is necessarily inefficient: the market equilibrium supports a stationary path with declining frictions, but the planner’s allocation does not. This inefficiency arises because the bargaining weight in the market equilibrium is fixed, whereas the planner’s shadow bargaining power varies with tightness, preventing the Hosios condition (Hosios, 1990) from being satisfied. Thus, while the biased-technology DMP framework restores the core qualitative properties of DMP models and avoids the implausible welfare implications of inspection, it reveals an intrinsic inefficiency between market and planner allocations.

Together, these results show that MM’s characterization overstates the conditions required for balanced growth. Pareto distributions and inspection are sufficient under a particular (multiplicative) time transformation, but not necessary once alternative transformations are considered. In this article, we focus on the two most transparent cases—multiplicative and additive detrending—but the logic extends to more general transformations of time that can also restore stationarity. Our analysis clarifies the logical structure of balanced growth in search models and provides a broader foundation for understanding how declining frictions interact with stationarity and growth.

The rest of the paper proceeds as follows. Section 2 revisits MM’s inspection model. Section 3 shows that with exponential sampling, a BGP exists where unemployment and vacancies remain stationary but declining frictions do not generate growth. Section 4 examines a DMP model with homogeneous workers and biased technological progress in the matching function, showing that a well-behaved BGP exists when unemployment and vacancies are complementary inputs. Section 5 concludes.

## 2 Overview

Because the paper is technically detailed, it is useful to begin with an overview of the main ideas. MM analyze a two-stage matching process in the labor market. In the first stage, a worker meets a vacancy with arrival rate  $A_t p(\theta_t)$ , where  $\theta_t$  is labor market tightness defined as vacancies over unemployed workers and  $A_t$  is a technological parameter growing exogenously at the constant rate  $g_A$ :  $A_t = A_0 e^{g_A t}$ . MM refer to this as the *meeting rate*. If a meeting occurs, the process moves to the second stage, where the productivity of the match is drawn from a distribution  $F(z)$ . The unemployment-employment (UE) rate is thus

$$h_{ue,t} = A_t p(\theta_t) (1 - F(R_t)), \quad (1)$$

where  $R_t$  is the reservation threshold of match quality.

MM seek a balanced growth path (BGP) in which both  $h_{ue}$  and  $\theta$  remain constant, consistent with the empirical stationarity of unemployment and vacancies. Since the matching technology improves over time, the reservation threshold must rise accordingly to satisfy the equation at all times. Differentiating yields

$$g_A = \frac{F'(R_t)}{1 - F(R_t)} \dot{R}_t, \quad \forall t, \quad (2)$$

and under the assumption that  $R_t$  grows at a constant rate, MM obtain their key condition (equation (10)), which implies that  $F$  must be Pareto. They further show that  $R_t$  indeed grows at a constant rate as an equilibrium outcome, and that increasing selectivity adds to economic growth.

Our paper builds on three observations that qualify and extend this framework.

**1. Pareto not necessary.** Their equation (10) is a special case of the more general condition, equation (2). While Pareto sampling produces a constant UE transition rate, other distributions, such as exponential, can do so as well once the adjustment of reservation quality is specified differently. For instance, if  $\dot{R}_t$  is constant (rather than  $\dot{R}/R_t$ ), the reservation quality increases linearly rather than exponentially over time, and the solution to the differential equation (2) is exponential rather than Pareto. With exponential sampling, unemployment, tightness, and transition rates remain stationary, but output growth slows down

and eventually stops. Thus, declining search frictions do not drive long-run growth, and stationarity does not uniquely require a Pareto distribution.

**2. Inspection models have counterfactual implications.** Introducing inspection fundamentally changes the standard DMP logic. In the canonical model, eliminating search frictions (making vacancies costless) drives unemployment to zero and yields finite welfare gains. In inspection models, by contrast, costless posting pushes tightness and reservation productivity to infinity. With Pareto sampling, both forces cancel each other out and unemployment persists even under costless posting. Moreover, the welfare gains from eliminating frictions become infinite, an implausible prediction.

**3. Technological progress may be biased.** MM assume Hicks-neutral progress, with  $A_t$  scaling the entire meeting rate. A more general formulation is

$$h_{ue,t} = p(\theta_t, A_t)(1 - F(R_t)),$$

where  $A_t$  affects the productivity of unemployment and vacancies differently. When these inputs are complements, biased progress in one input eventually faces diminishing returns: improvements in  $A_t$  no longer raise the meeting rate without bound, and the need for ever-rising reservation thresholds disappears. In the limit, search efficiency gains no longer fuel growth, undermining MM's claim that declining frictions necessarily generate long-run expansion.

### 3 Inspection

This section considers variants of the MM inspection model, with the aim of characterizing the necessary and sufficient conditions for the existence of a suitable balanced growth path. While the original framework analyzes a decentralized equilibrium, we study the corresponding social planner's allocation and show that the two coincide if the Hosios condition holds and the matching function is Cobb-Douglas. Our formulation generalizes the baseline model to incorporate both endogenous and exogenous job destruction, nesting the canonical case without declining search frictions as a special case.

#### 3.1 Environment

The economy is populated by a continuum of workers of measure one and a continuum of firms of positive measure. At each date  $t$ , each worker is either unemployed,  $u_t$ , or employed in a job with firm-specific productivity  $z$ . Let  $n_t(z)$  denote the measure of workers at time  $t$  employed in jobs with productivity  $z$ . Employed workers with productivity  $z$  produce  $y_t z$ , where  $y_t = y_0 e^{g_y t}$  is an aggregate productivity term common to all jobs and grows at rate  $g_y \geq 0$ . Jobs are destroyed at an exogenously rate  $\delta \geq 0$ , or endogenously when either the worker or the firm chooses to separate. In the planner's allocation, endogenous

separations are characterized by a productivity cutoff  $R_t$ : matches with  $z < R_t$  are terminated, while those with  $z \geq R_t$  continue. Unemployed workers produce  $b_t$ .

Unemployed workers can be assigned to jobs across different productivity levels, but doing so requires vacancy creation. Let  $v_t$  denote vacancies posted at cost  $k_t$  units of output per vacancy per period. These vacancies generate  $A_t M(u_t, v_t)$  random matches between unemployed workers and vacant jobs, where  $A_t$  measures search efficiency, and  $M$  is a constant-returns-to-scale (CRS) matching function.  $M$  is increasing in each argument, concave, and satisfies the Inada conditions. Let  $M_1$  and  $M_2$  denote the corresponding partial derivatives with respect to the first and second arguments, respectively.

When a worker and firm meet, a match productivity  $z$  is drawn from a cumulative distribution  $F(z)$  with density  $f(z)$  and support  $[z_l, \infty)$ . The law of motion of  $n_t(z)$  is:

$$\dot{n}_t(z) = A_t M(u_t, v_t) f(z) - \delta n_t(z) \text{ for } z \geq R_t \text{ and } t \geq 0.$$

### 3.2 Planner's Problem

Given an initial distribution of employment,  $[n_0(z)]_0^\infty$ , the planner solves the following problem:

$$\max_{\{v_t, u_t, R_t, n_t\}_{t=0}^\infty} \int_{t=0}^\infty e^{-rt} \left[ \int_{R_t} y_t z n_t(z) dz + u_t b_t - k_t v_t \right] dt \text{ subject to:}$$

$$\dot{n}_t(z) = A_t M(u_t, v_t) f(z) - \delta n_t(z) \text{ for } z \geq R_t \text{ for } t \geq 0, \text{ and} \quad (3)$$

$$u_t = 1 - \int_{R_t}^\infty n_t(z) dz, \quad R_t \geq z_l \text{ for } t \geq 0. \quad (4)$$

The planner selects vacancies, unemployment, and a productivity cutoff to maximize the discounted present value of net output at discount rate  $r$ , subject to the law of motion for employment across job productivities and the labor market resource constraint.

To connect this formulation with the MM decentralized equilibrium, it is useful to define the meeting rates implied by the matching function. An unemployed worker meets a vacancy at rate  $m_t = A_t p(\theta_t)$  where  $p(\theta) \equiv M(1, \theta)$  and  $\theta_t = v_t/u_t$  is labor market tightness. Symmetrically, a vacancy meets an unemployed worker at rate  $s_t = A_t q(\theta_t)$ , where  $q(\theta) = p(\theta)/\theta$ .

### 3.3 Optimality Conditions

Let  $e^{-rt} \lambda_t(z)$  and  $e^{-rt} \eta_t$  be the Lagrange multipliers associated with equation (3) and equation (4), respectively, for  $t \geq 0$ . The conditions associated with the optimal choices of  $v_t$ ,  $u_t$ ,  $R_t$ , and  $n_t(z)$  are:

$$k_t = A_t M_2(1, \theta) \int_{R_t}^\infty \lambda_t(z) f(z) dz = A_t q_t (1 - \mu_t) \int_{R_t}^\infty \lambda_t(z) f(z) dz, \quad (5)$$

$$\eta_t = b_t + A_t M_1(1, \theta) \int_{R_t} \lambda_t(z) f(z) dz = b_t + A_t p_t \mu_t \int_{R_t} \lambda_t(z) f(z) dz, \quad (6)$$

$$\eta_t = y_t R_t, \text{ and} \quad (7)$$

$$y_t z - \eta_t - \delta \lambda_t(z) = r \lambda_t(z) - \dot{\lambda}_t(z) \text{ for } z \geq R_t, \quad (8)$$

where  $\mu_t$  is the elasticity of matches with respect to unemployment.<sup>2</sup>

Equation (5) states that the optimal mass of vacancies equates the marginal cost  $k_t$  to the marginal benefit: the additional matches created by an extra vacancy,  $A_t \frac{\partial M_t}{\partial v_t} = A_t q_t (1 - \mu_t)$ , multiplied by the expected shadow value of a filled job.

Equation (6) requires that the shadow flow value of an unemployed worker,  $\eta_t$ , equals the flow of output while unemployed,  $b_t$ , plus the additional matches generated by an unemployed worker,  $A_t \frac{\partial M_t}{\partial u_t} = A_t p_t \mu_t$ , times expected shadow value of a match.

Equation (7) implies that the optimal reservation productivity  $R_t$  is such that the production of the marginal worker,  $y_t R_t$ , equals the shadow flow value of unemployment,  $\eta_t$ . Finally, equation (8) characterizes the shadow value  $\lambda_t(z)$  of a filled job.

We now rewrite these expressions in a form that will be useful later. Combining equation (6) and equation (7) yields:

$$y_t R_t - b_t = p_t \mu_t \int_{R_t} \lambda_t(z) f(z) dz.$$

Substituting equation (5) into this expression leads to our first key relationship:

$$\frac{y_t R_t - b_t}{k_t} = \frac{p_t \mu_t}{q_t (1 - \mu_t)} = \frac{\mu(\theta_t)}{1 - \mu(\theta_t)} \theta_t. \quad (9)$$

Next, substituting equation (7) into equation (8) gives:

$$(r + \delta) \lambda_t(z) = y_t z - y_t R_t + \dot{\lambda}_t(z) \text{ for } z \geq R_t. \quad (10)$$

This is the familiar value function equation in which  $\lambda_t(z)$  denotes the social value of a match. Solving this differential equation (see Appendix) yields:

$$\lambda_t(z) - e^{-(r+\delta)d} \lambda_{t+d}(z) = \int_0^d e^{-(r+\delta)\tau} (y_{t+\tau} z - y_{t+\tau} R_{t+\tau}) d\tau. \quad (11)$$

The transversality condition  $e^{-(r+\delta)d} \lambda_{t+d} = 0$  must hold. If  $d$  is finite, this implies  $\lambda_{t+d} = 0$  or, from equation (10),

$$z = R_{t+d_t}(z). \quad (12)$$

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<sup>2</sup>We assume that parameters are such that optimal solutions are interior. We discuss some of these restrictions below in the context of specific sampling distributions,  $F$ .

Equation (12) defines  $d_t(z)$ , the optimal longevity of a match with productivity  $z$  at time  $t$ , absent an exogenous destruction shock. Substituting this into equation (11) gives:

$$\lambda_t(z) = \int_0^{d(z,t)} e^{-(r+\delta)\tau} (y_{t+\tau}z - y_{t+\tau}R_{t+\tau}) d\tau. \quad (13)$$

### 3.4 Aggregates

The measure of aggregate employment with match quality below  $z_t$  is:

$$N_t(z_t) \equiv \int_{R_t}^{z_t} n_t(x) dx. \quad (14)$$

Total employment is then:

$$N_t = N_t(\infty) = 1 - u.$$

From equation (3) and equation (14), we obtain:

$$\begin{aligned} \dot{N}_t(z_t) &= n_t(z_t)\dot{z}_t - n_t(R_t)\dot{R}_t + \int_{R_t}^{z_t} \dot{n}_t(x) dx \\ &= n_t(z_t)\dot{z}_t - n_t(R_t)\dot{R}_t + \int_{R_t}^{z_t} [A_t M(u_t, v_t) f(x) - \delta n_t(x)] dx \\ &= n_t(z_t)\dot{z}_t - n_t(R_t)\dot{R}_t + u_t A_t p_t (F(z_t) - F(R_t)) - \delta N_t(z_t). \end{aligned} \quad (15)$$

This expression decomposes employment below  $z_t$  into net inflows from new matches and outflows from job destructions.

Similarly, the law of motion for total employment can be expressed as:

$$\begin{aligned} \dot{N}_t &= \int_{R_t}^{\infty} \dot{n}_t(z) dz - n_t(R_t)\dot{R}_t \\ &= u_t A_t p_t (1 - F(R_t)) - \delta N_t - n_t(R_t)\dot{R}_t. \end{aligned} \quad (16)$$

Here, total employment depends on the inflow from unemployed workers matched to jobs above  $R_t$  and the outflow from job destruction. Finally, define the cumulative distribution of match qualities as:

$$G_t(z) \equiv \frac{N_t(z)}{N_t} \quad (17)$$

This is the fraction of employed workers in matches with productivity below  $z$ .

### 3.5 Balanced Growth

#### 3.5.1 Definition

The definition of a balanced growth path is central, as it imposes additional restrictions on endogenous variables, thereby freeing up equations that can be



used to pin down the exogenous forces—particularly the dynamics of  $k_t$ ,  $b_t$ , and the function  $F$  — needed to sustain the path. We consider two definitions, denoted by BPG and SBGP, both employed by MM.

**Definition:** A *Balanced Growth Path* (BGP) is an initial state,  $G_0(z)$ , and associated efficient allocation such that unemployment, tightness, employment-to-unemployment (EU), and unemployment-to-employment (UE) rates are constant over time while aggregate productivity and search technologies improve at constant rates,  $g_y \geq 0$  and  $g_A \geq 0$ . A *Scaled Balanced Growth Path* (SBGP) is a BGP in which the distribution of employed workers across match qualities,  $G_t(z)$ , shifts at a constant rate:

$$G_t(ze^{g_z t}) = G_0(z) \text{ for all } z \geq R_0.$$

By construction, an SBGP is a special case of a BGP—one that imposes a specific proportional (multiplicative) transformation on the endogenous distribution  $G_t$ . MM describe the broader BGP concept in the abstract and introduction of their paper, but adopt the more restrictive SBGP definition in proving their main result, Theorem 1. The BGP definition, in contrast, places no restriction on the evolution of  $G_t$  beyond requiring that aggregate labor-market variables remain stationary over time. This broader interpretation is fully consistent with MM’s stated objective in their abstract: “*We seek conditions for the existence of a balanced growth path (BGP), where unemployment, vacancy, and worker’s transition rates remain constant in the face of improvements in the production and search technologies.*”

The restriction MM impose on  $G_t$  is inspired by related work (e.g., Perla and Tonetti, 2014; Lucas and Moll, 2014; Buera and Oberfield, 2020; Benhabib et al., 2021), which adopts a similar assumption for models of endogenous growth. In that context, the restriction is natural and purposeful as the restriction generates endogenous growth. In the present context—where the aim is to explain labor market statistics—it inadvertently introduces endogenous growth, even though it is not required for the stated objective. One implication of this definition is that the reservation productivity, the lower bound of the support, must grow at the constant rate  $g_z$  along a SBGP:

$$R_t = R_0 e^{g_z t}.$$

The BGP definition does not impose this structure on the reservation productivity, and in fact our counter-example instead requires  $R_t = R_0 + \phi \times t$  where  $\phi > 0$  is a constant.

Finally, the BGP definition permits  $g_A \geq 0$ , whereas MM require  $g_A > 0$ . Their restriction rules out the canonical benchmark of stable search frictions (constant  $A$ ). The reason is that in their framework, when  $A$  is constant, the distribution  $F$  of match quality cannot be identified. Moreover, because technological progress in search is the sole source of job destruction in MM, setting  $g_A = 0$  renders employment an absorbing state. By contrast, our framework introduces exogenous job destruction ( $\delta > 0$ ), allowing the case  $g_A = 0$  to be well defined and to place no restrictions on  $F$ .

### 3.6 BGP and SBGP System of Equations

Along a BGP, the following versions of equations (5), (9), (12), (13), (15), and (16) must hold. First, the first order condition with respect to vacancies becomes

$$k_t = A_t M_2(1, \theta) \int_{R_t} \lambda_t(z) f(z) dz, \quad (18)$$

while the reservation condition reads

$$\frac{y_t R_t - b_t}{k_t} = \frac{M_1(1, \theta)}{M_2(1, \theta)} = \frac{\mu(\theta)}{1 - \mu(\theta)} \theta. \quad (19)$$

Match longevity is defined by

$$z = R_{t+d_t(z)}, \quad (20)$$

and the shadow value of a filled job satisfies

$$\lambda_t(z) = \int_0^{d(z)} e^{-(r+\delta)\tau} (y_{t+\tau} z - y_{t+\tau} R_{t+\tau}) d\tau. \quad (21)$$

The dynamics of employment by productivity are given by

$$n_t(z_t) \dot{z}_t + u A_t p(\theta) (F(z_t) - F(R_t)) = n_t(R_t) \dot{R}_t + \delta N_t(z_t). \quad (22)$$

In steady state, inflows into unemployment must equal outflows, implying

$$u h_{ue} = (1 - u) h_{eu}, \quad (23)$$

where the job-finding and job-destruction rates are given by:

$$h_{ue} \equiv A_t p(\theta) (1 - F(R_t)), \quad (24)$$

$$h_{eu} \equiv \delta + \frac{n_t(R_t)}{N} \dot{R}_t, \quad (25)$$

with total employment  $N = 1 - u$ .

Given parameters, these conditions can be used to solve for  $d_t(z)$ ,  $R_t$ ,  $\theta$ ,  $\lambda_t(z)$ ,  $n_t(z)$ ,  $h_{ue}$ , and  $h_{eu}$ . Equation (22) corresponds to equation (15) when  $\dot{N}_t(z_t) = 0$ , and equation (23) corresponds to equation (16) when  $\dot{N}_t = 0$ . Finally, equation (24) and equation (25) provide explicit definitions for the steady-state job-finding and job-destruction rates.

As in MM, it is convenient to work with the distribution  $G_t$  defined in equation (17). To derive the implied restrictions on  $G$ , substitute equations (23)-(24) into equation (22):

$$\begin{aligned} n_t(z_t) \dot{z}_t + u A_t p(\theta) (F(z_t) - F(R_t)) &= h_{eu} N - \delta N + \delta N_t(z_t) \\ &= u h_{ue} + \delta (N_t(z_t) - N) \\ &= u A_t p(\theta) (1 - F(R_t)) + \delta (N_t(z_t) - N). \end{aligned}$$

Simplifying yields:

$$n_t(z_t)\dot{z}_t = uA_t p(\theta)(1 - F(z_t)) + \delta(1 - u) \left( \frac{N_t(z_t)}{N} - 1 \right).$$

From equation (17) we have that  $(1 - u)G'_t(z) = n_t(z)$ . Substituting this into the previous expression gives:

$$(1 - u) \left( G'_t(z_t)\dot{z}_t + \delta(1 - G_t(z_t)) \right) = uA_t p(\theta)(1 - F(z_t)), \quad (26)$$

which provides the implied restriction on the evolution of the endogenous employment distribution. Moreover, the job destruction rate (25) becomes:

$$h_{eu} = \delta + G'_t(R_t)\dot{R}_t. \quad (27)$$

These last two equations can replace equation (22) and equation (25) in the definition of BGP. On a Scaled Balanced Growth Path (SBGP), productivity grows at a constant rate  $\dot{z}_t/z_t = g_z$ . Substituting this into equation (26) and equation (27) yields

$$(1 - u) (G'_t(z_t)z_t g_z + \delta(1 - G_t(z_t))) = uA_t p(\theta)(1 - F(z_t)), \quad (28)$$

$$h_{eu} = \delta + G'_t(R_t)R_t g_z. \quad (29)$$

These two conditions replace equation (26) and equation (27) in the definition of a SBGP.

Finally, average match quality satisfies

$$Z_t = \int_{R_t} z G'_t(z) dz, \quad (30)$$

which links the distribution of employed matches to aggregate productivity.

**Decentralization and efficiency.** Simple inspection reveals the connection between the social planner's solution and the decentralized equilibrium of MM. Our system of equations defining a SBGP is identical to their system if  $\delta = 0$  (i.e., all job separations are endogenously determined by the reservation rule) and the matching elasticity  $\mu(\theta)$  is substituted by a constant workers' bargaining power,  $\gamma$ . In other words, when the Hosios condition holds and the matching function is Cobb-Douglas, MM's decentralized equilibrium is efficient.

### 3.7 Characterization

We now present three main results. The first two parallel MM's Lemma 1 and Theorem 1, which establish necessary and sufficient conditions for the existence of a SBGP. Our results are novel in that they pertain to efficient rather than decentralized equilibrium allocations and allow for both endogenous and exogenous separations, whereas MM consider only the endogenous separations.

The third result is a counterexample: a BGP that is not a SBGP, where the distribution  $F$  is exponential rather than Pareto, yet labor market statistics remain constant over time. This example demonstrates that search models need not be constrained by the narrow conditions of MM's Lemma 1 and Theorem 1.

**Lemma 1 (Necessary conditions for a SBGP).** Let  $g_y \geq 0$  and  $g_A > 0$  be arbitrary growth rates for the production and search technologies.

1. A SBGP may exist only if (a) the distribution  $F$  is Pareto with an arbitrary coefficient  $\alpha$ ; (b) the growth rate of the vacancy cost,  $g_k$ , and the growth rate of the unemployment benefit,  $g_b$ , are equal to  $g_y + g_z$ ; and (c) the discount rate  $r$  is greater than  $g_y + g_z$ .
2. In any SBGP, the growth rate  $g_z$  of the distribution  $G_t$  is equal to  $g_A/\alpha$ .

The proof of Lemma 1 follows MM closely and is therefore omitted. The need for a Pareto distribution follows from our discussion in Section 2. Intuitively, a Pareto distribution is required to ensure scale invariance of match quality, so that the reservation cutoff can grow proportionally over time. Moreover, equation (12) in MM, needed for the proof, is analogous to our equation (19), except that our expression involves the endogenous elasticity  $\mu(\theta)$  rather than the exogenous bargaining weight  $\gamma$ . But since  $\theta$  is constant along a BGP, this difference does not affect the proof of Lemma 1.

Assuming that the conditions of Lemma 1 are satisfied, the next step is to show that a SBGP exists and is unique. This amounts to verifying that, under those conditions, the system of equations defining a SBGP can be solved and yields a unique set of functions and values for  $d_t(z)$ ,  $R_t$ ,  $\theta$ ,  $\lambda_t(z)$ ,  $G_t(z)$ ,  $h_{ue}$ , and  $h_{eu}$ . Following MM's steps, this system can be reduced to two equations in the two unknowns  $R_0$  and  $\theta$ :

$$R_0 = (A_0 M_2(1, \theta) \Phi_1 y_0 / k_0)^{1/(\alpha-1)}, \quad (31)$$

$$R_0 = \frac{b_0}{y_0} + \frac{M_1(1, \theta) k_0}{M_2(1, \theta) y_0}, \quad (32)$$

where

$$\Phi_1 = \frac{z_l^\alpha}{(\alpha-1)((\alpha-1)g_z + (r + \delta - g_y))}. \quad (33)$$

Given the assumed properties of  $M$ , this system has a unique solution. Equation (31) is a strictly decreasing function of  $\theta$ , spanning from  $+\infty$  to 0, while equation (32) is a strictly increasing function of  $\theta$ , spanning from 0 to  $+\infty$ . By the intermediate value theorem, the two curves intersect at a unique  $\theta$ , which binds down a unique solution for  $R_0$  and  $\theta$ . The solution for  $R_t$  is then  $R_t = R_0 e^{(g_A/\alpha)t}$ . A unique solution for  $G_0(z)$  can then be found.

Given that  $F$  is Pareto, equation (28) reads:

$$(1-u)(G'_t(z)zg_z + \delta(1-G_t(z))) = uA_t p(\theta) \left(\frac{z_l}{z}\right)^\alpha.$$

This is a differential equation for  $G_t(z)$ . A natural guess for the solution is  $G_t(z) = 1 - \left(\frac{R_t}{z}\right)^\alpha$ . Substituting this into the differential equation yields

$$(1 - u) \left(\frac{R_t}{z}\right)^\alpha (\alpha g_z + \delta) = u A_t p(\theta) \left(\frac{z_l}{z}\right)^\alpha.$$

This equation is satisfied if

$$(1 - u) (\alpha g_z + \delta) = u A_t p(\theta) \left(\frac{z_l}{R}\right)^\alpha = u A_t p(\theta) (1 - F(R)) = u h_{ue},$$

or

$$u = \frac{\alpha g_z + \delta}{\alpha g_z + \delta + h_{ue}} = \frac{g_A + \delta}{g_A + \delta + h_{ue}}.$$

This condition is consistent with equation (23) and equation (29), together with the guessed solution for  $G_t(z)$ , thereby confirming its validity. Hence, an efficient SBGP exists, is unique and—importantly—is continuous at  $g_A = 0$  if  $\delta > 0$ . This continuity is possible because  $\delta > 0$  allows for exogenous job destruction. By contrast, if  $\delta = 0$  and  $g_A = 0$ , there would be no job destruction along a SBGP making full employment an absorbing state. The following Theorem collects these results.

**Theorem 1 (Existence and properties of a SBGP).** Let  $g_A > 0$ ,  $g_y \geq 0$ , and  $\delta > 0$ . An (interior) efficient SBGP exists if and only if (a)  $F$  is Pareto with coefficient  $\alpha > 1$ ; (b)  $g_b$  and  $g_k$  satisfy  $g_b = g_y + g_A/\alpha$  and  $g_k = g_y + g_A/\alpha$ ; and (c)  $r > g_y + g_A/\alpha$ .

If an (interior) efficient SBGP exists, it is unique, continuous at  $g_A = 0$ , and has the following properties:

- (i)  $u$ ,  $\theta$ ,  $h_{ue}$ , and  $h_{eu}$  are constant, with  $h_{ue} = A_0 p(\theta) (1 - F(R_0))$ ,  $h_{eu} = g_A + \delta$ ;
- (ii)  $G_t(z e^{g_z t}) = G_0(z)$  with  $g_z = g_A/\alpha$  and  $G_0(z) = 1 - \left(\frac{R_0}{z}\right)^\alpha$ ; and
- (iii) labor productivity grows at the rate  $g_y + g_A/\alpha$ .

The theorem confirms MM's findings for the planner. Under DMP decentralization, efficient and market allocations generally differ, except when the Hosios condition holds and the matching function is Cobb-Douglas. In contrast to MM, our SBGP is continuous at  $g_A = 0$ .

Theorem 1 includes condition (d), which is required to ensure that first-order conditions are necessary. MM does not discuss the possibility of corner solutions, but as we discuss in Section 3.8, corner solutions may arise if the Pareto tail is too fat.

We now present the main result of this subsection: an example of a BGP that is not an SBGP. As discussed in Section 2, when the sampling distribution is exponential rather than Pareto, maintaining a constant UE rate requires the reservation productivity to increase linearly over time rather than exponentially, ensuring that declining search frictions do not contribute to long-run growth. The limit of our BGP example is therefore an SBGP without growth. The

following proposition shows that this outcome arises, as a general equilibrium result, when unemployment benefits rise linearly over time while the cost of posting a vacancy remains constant.

**Proposition 1 (BGP)** *Let  $y = 1$ ,  $g_A > 0$ ,  $F(z) = 1 - e^{-\lambda z}$ ,  $k_t = k$ , and  $b_t = b + (g_A/\lambda)t$ . Then there exists a unique efficient BGP such that:*

- (i)  $u$ ,  $\theta$ ,  $h_{ue}$ , and  $h_{eu}$  are constant, with  $h_{ue} = A_0 p(\theta)(1 - F(R_0))$ ,  $h_{eu} = g_A + \delta$ ;
- (ii)  $R_t = R_0 + (g_A/\lambda)t$ ;
- (iii)  $G_t(z + (g_A/\lambda)t) = G_0(z) = 1 - e^{-\lambda(z-R_0)}$ ;
- (iv) average productivity satisfies  $Z_t = 1/\lambda + R_t$ . Hence

$$\frac{\dot{Z}_t}{Z_t} = \frac{g_A}{1 + \lambda R_0 + g_A t} \rightarrow 0 \quad \text{as } t \rightarrow \infty.$$

**Proof.** Following MM's steps, the system reduces to two equations in the two unknowns  $R_0$  and  $\theta$ . Equation (24) becomes

$$h_{ue} = A_0 e^{g_A t} p(\theta) e^{-\lambda R_t} = A_0 p(\theta) e^{-\lambda R_0}.$$

From this expression, it follows that

$$R_t = R_0 + \frac{g_A}{\lambda} t. \quad (34)$$

Substituting into equation (19) yields the first equation in two unknowns:

$$\frac{R_t - b_t}{k} = \frac{R_0 - b}{k} = \frac{M_1(1, \theta)}{M_2(1, \theta)}. \quad (35)$$

Next, using equation (34) in equation (20) gives  $d(z) = (z - R_t) / (g_A/\lambda)$ . The appendix shows that

$$\int_{R_t} \lambda_t(z) f(z) dz = \Phi_2 e^{-\lambda R_t}, \quad (36)$$

where  $\Phi_2 = \frac{1}{r+\delta} \frac{1}{\lambda} \left[ \frac{r+\delta-g_A}{r+\delta} + \frac{1}{r+\delta} \frac{g_A^2}{r+\delta+g_A} \right]$ . Plugging this and equation (34) into equation (18) gives the second equation:

$$k = A_t e^{-\lambda R_t} M_2(1, \theta) \Phi_2 = A_0 e^{-\lambda R_0} M_2(1, \theta) \Phi_2. \quad (37)$$

Equations (35) and (37) determine  $\theta$  and  $R_0$ . Given the assumed properties of  $M$ , this system admits a unique solution. Now consider equation (26):

$$(1 - u) \left( G'_t(z) \frac{g_A}{\lambda} + \delta (1 - G_t(z)) \right) = u A_t p(\theta) e^{-\lambda z}.$$

This is a differential equation for  $G_t(z)$ . Given the exponential form on the right-hand side, a natural conjecture is

$$G_t(z) = 1 - e^{-\lambda(z-R_t)}.$$

Substituting yields

$$(1 - u) e^{-\lambda(z - R_t)} (g_A + \delta) = u A_t p(\theta) e^{-\lambda z}.$$

This holds if and only if

$$(1 - u) (g_A + \delta) = u A_t p(\theta) e^{-\lambda R_t} = u A_t p(\theta) (1 - F(R_t)) = u h_{ue},$$

or equivalently,

$$u = \frac{g_A + \delta}{g_A + \delta + h_{ue}} = \frac{g_A + \delta}{g_A + \delta + A_t p(\theta) e^{-\lambda R_t}},$$

which follows from equations (23), (24), (27), and the conjecture for  $G$ . Thus, the conjecture is verified. Finally, average match quality—equal to average labor productivity since  $y = 1$ —defined in equation (30) satisfies

$$Z_t = \frac{1}{\lambda} + R_t.$$

Hence,

$$\frac{\dot{Z}_t}{Z_t} = \frac{\dot{R}_t}{Z_t} = \frac{g_A}{1 + \lambda R_0 + g_A t} \rightarrow 0.$$

■

Comparing Theorem 1 and Proposition 1, we find that if the sample distribution is Pareto, a SBGP requires both the cost of posting a vacancy and unemployment compensation to rise exponentially over time. By contrast, if the sample distribution is exponential, the cost of posting a vacancy can remain constant, while unemployment compensation needs to grow only linearly. The underlying reason is that under an exponential distribution, small changes in the reservation threshold have large effects on the probability of drawing a high-quality match. Consequently, parameters do not need to adjust as much to sustain a BGP.

In summary, MM imposes an unnecessary extra assumption. They assume that in any BGP, the reservation productivity must grow at a constant rate. Proposition (1) removes this assumption. While it is true that an increase in reservation productivity is necessary to offset the persistent improvements in search technology, this increase does not need to be exponential over time.

### 3.8 The Role of Search Frictions

We now turn to the implications of eliminating search frictions. In the canonical DMP model, removing frictions—by making vacancy posting costless—drives unemployment to zero. In the MM inspection framework, by contrast, unemployment does not vanish: although jobs become easy to find, workers simultaneously become excessively selective, exactly offsetting the improved matching

prospects. In our exponential benchmark, however, unemployment does vanish, since self-selection is too weak to neutralize the ease of finding work.

The welfare predictions differ just as sharply. In the DMP model, the gains from eliminating frictions are finite, bounded by the additional output that unemployed workers could contribute. In both the MM model and our exponential inspection variant, however, the welfare gains are unbounded. As matching becomes arbitrarily easy, it is optimal for workers to hold out for ever-better offers, and since match quality is unbounded above, expected gains diverge. This implausible implication highlights a core weakness of inspection frameworks. The present section illustrates these weaknesses, while Section 4 develops an alternative mechanism that avoids them.

For this analysis, we adopt the canonical Cobb-Douglas (CD) matching function,  $M(v, u) = u^\gamma v^{1-\gamma}$ , to facilitate comparison with MM's formulas.

### 3.8.1 The Underlying Source of Unemployment

In the two models we are considering, the unemployment rate satisfies

$$u = \frac{g_A + \delta}{g_A + \delta + h_{ue}}, \quad (38)$$

where  $h_{ue}$  satisfies equation (24). In this section, we are interested in the limit of this expression as  $k \rightarrow 0$ . In the DMP model, free vacancy posting would drive vacancies, tightness,  $\theta$ , workers' matching probability, and the UE transition probability,  $h_{ue}$ , to infinite, thus eliminating unemployment.

### 3.8.2 MM's Model

In the MM model, the UE transition satisfies

$$h_{ue} \equiv A_0 p(\theta) (1 - F(R_0)) = A_0 \theta^{1-\gamma} z_l^\alpha R_0^{-\alpha}. \quad (39)$$

In order to characterize  $h_{ue}$ , we need to characterize  $\theta$  and  $R_0$ .

Using the assumed matching function, equation (31) and equation (32) simplify to:

$$R_0 = \left( A_0 (1 - \gamma) \Phi_1 \frac{y_0}{k_0^{1-\gamma}} \right)^{1/(\alpha-1)} (\theta k_0)^{-\gamma/(\alpha-1)}, \text{ and} \quad (40)$$

$$R_0 = \frac{b_0}{y_0} + \frac{\gamma}{1 - \gamma} \frac{\theta k_0}{y_0}. \quad (41)$$

Equation (40) depicts  $R_0$  as a decreasing function of  $\theta k_0$  while equation (41) depicts  $R_0$  as an increasing function of  $\theta k_0$ . A unique solution for  $R_0$  and  $\theta k_0$  exists. Moreover, as  $k_0$  decreases, the curve from equation (40) shifts upwards and that from equation (41) remains unchanged, pushing the solution toward higher  $R_0$  and  $\theta k_0$ , but also higher  $\theta$ . As  $k \rightarrow 0$  both  $R_0$  and  $\theta$  diverge to  $\infty$ .



For  $k$  sufficiently low, the constant term  $\frac{b_0}{y_0}$  in equation (41) becomes negligible so that

$$R_0 \approx \frac{\gamma}{1-\gamma} \frac{\theta k_0}{y_0}.$$

Using this and equation (40), closed form solution for  $R_0$  and  $\theta$  can be found as:

$$\theta_{k \rightarrow 0} = \Omega_1 k_0^{-\frac{\alpha}{\alpha-(1-\gamma)}}, \text{ and} \quad (42)$$

$$R_{k \rightarrow 0} = \frac{\gamma}{1-\gamma} \frac{\Omega_1}{y_0} k_0^{-\frac{1-\gamma}{\alpha-(1-\gamma)}}, \quad (43)$$

where  $\Omega_1 = \left[ \frac{1-\gamma}{\gamma} (A_0 (1-\gamma) \Phi_1)^{1/(\alpha-1)} y_0^{\alpha/(\alpha-1)} \right]^{\frac{1}{1+\gamma/(\alpha-1)}}$ . These solutions are valid in the limit, but if  $b_0 = 0$ , they are also valid for any  $k$ . Substituting these solutions into equation (39), the solution for  $h_{ue}$  is obtained. The solution then can be substituted into equation (38) to find unemployment. The following proposition summarizes the main results.

**Proposition 2** *In a BGP with  $M(u, v) = u^\gamma v^{1-\gamma}$ , and  $k \rightarrow 0$ ,*

$$h_{ue} \rightarrow h_{ue}^* = (\alpha - 1) \left( \frac{\alpha - 1}{\alpha} g_A + r + \delta - g_y \right) / \gamma,$$

$$u^* = \frac{g_A + \delta}{g_A + \delta + h_{ue}^*} > 0.$$

**Proof.** See Appendix. ■

**Discussion.** Unlike the DMP model, unemployment persists in MM's model even when posting vacancies is costless. Firms post infinitely many vacancies, making matches certain, but workers become increasingly selective and wait for the best possible draws. Consequently, unemployment remains strictly positive and independent of level parameters such as  $A_0$ ,  $z_l$ , or  $k$ . Moreover, unemployment decreases with the Pareto tail parameter: thinner tails induce workers to accept jobs more readily. The key driver of long run unemployment in MM's model is the random nature of match quality. If  $\alpha = \infty$ , so that match quality is deterministically  $z_l$ , then unemployment would fall to zero when  $k = 0$ .

### 3.8.3 BGP with Exponential Sample Distribution

The outcome changes when the sample distribution is exponential rather than Pareto. Because the exponential distribution has much thinner tails, the selection motive is too weak to sustain positive unemployment in the presence of abundant job offers. As a result, unemployment vanishes once vacancies become costless, as we show.

Under the exponential distribution and CD matching, the UE transition rate is

$$h_{ue} = A_0 p(\theta) e^{-\lambda R_0} = A_0 \theta^{1-\gamma} e^{-\lambda R_0}.$$

To determine  $h_{ue}$ , we solve for  $(R_0, \theta)$  using the efficient conditions (35) and (37). Under CD matching, they become:

$$k = A_0 e^{-\lambda R_0} (1 - \gamma) \theta^{-\gamma} \Phi_2, \text{ and} \quad (44)$$

$$\theta = \frac{1 - \gamma}{\gamma} \frac{R_0 - b_0}{k}. \quad (45)$$

Substituting equation (45) into equation (44) gives a single equation in  $R_0$ :

$$LHS(R_0) := \frac{e^{-\lambda R_0}}{(R_0 - b_0)^\gamma} = \frac{k^{1-\gamma}}{A_0 (1 - \gamma)^{1-\gamma} \gamma^\gamma \Phi_2}. \quad (46)$$

Equation (46) admits a unique solution since  $LHS(b_0) = \infty$ ,  $LHS(\infty) = 0$ , and  $LHS'(R_0) < 0$ . Moreover, as  $k \rightarrow 0$ ,  $R_0 \rightarrow \infty$ . Once  $R_0$  is determined,  $\theta$  follows from equation (45).

Rewriting equation (44),

$$k = A_0 e^{-\lambda R_0} (1 - \gamma) \theta^{-\gamma} \Phi_2 = h_{ue} (1 - \gamma) \theta^{-1} \Phi_2,$$

which implies, using equation (45),

$$h_{ue} = \frac{k\theta}{(1 - \gamma) \Phi_2} = \frac{k \frac{1-\gamma}{\gamma} \frac{R_0 - b_0}{k}}{(1 - \gamma) \Phi_2} = \frac{R_0 - b_0}{\gamma \Phi_2}.$$

Thus the UE rate is increasing in the reservation quality  $R_0$ . Since  $\lim_{k \rightarrow 0} R_0 = \infty$ , it follows that  $h_{ue} \rightarrow \infty$ . Consequently, unemployment converges to zero.

### 3.8.4 Welfare Cost of Search Frictions

We now highlight a problematic feature of inspection models: eliminating search frictions by making vacancy posting costless leads to unbounded productivity and infinite welfare gains. Using equation (35) and CD matching, social welfare can be written as

$$\begin{aligned} W &= \int_{t=0}^{\infty} e^{-rt} \left[ \int_{R_t} y_t z n_t(z) dz + u b_t - k_t v \right] dt \\ &= \int_{t=0}^{\infty} e^{-rt} \left[ (1 - u) y_t \int_{R_t} z_t g_t(z) dz + u b_t - k_t \theta u \right] dt \\ &= \int_{t=0}^{\infty} e^{-rt} \left[ (1 - u) y_t Z_t + u b_t - \frac{1 - \gamma}{\gamma} (y_t R_t - b_t) u \right] dt, \end{aligned} \quad (47)$$

where  $g_t(z)$  is the density corresponding to the distribution function  $G_t(z)$ .

In the MM model, Lemma 1 and Proposition 1 yield the closed-form expression

$$\begin{aligned} W &= \int_{t=0}^{\infty} e^{-(r - g_y - g_z)t} \left[ (1 - u) \frac{\alpha}{\alpha - 1} y_0 R_0 + u b_0 - \frac{1 - \gamma}{\gamma} (y_0 R_0 - b_0) u \right] dt \\ &= \frac{\left[ (1 - u) \frac{\alpha}{\alpha - 1} - \frac{1 - \gamma}{\gamma} u \right] y_0 R_0 + \frac{1}{\gamma} u b_0}{r - g_y - g_z}. \end{aligned} \quad (48)$$

Since  $R_0 \rightarrow \infty$  as  $k \rightarrow 0$ , welfare diverges to  $+\infty$  provided the coefficient on  $y_0 R_0$  is positive—that is, when unemployment is not too high. Conditions (a) and (c) in Theorem 1 guarantee that it is, in fact, the case.

**Proposition 3** *In an interior SBGP with  $M(u, v) = u^\gamma v^{1-\gamma}$ ,  $\lim_{k \rightarrow 0} W = \infty$ . **Proof.** See Appendix. ■*

The implication of Proposition 3 is stark: eliminating search frictions delivers infinite welfare gains.

In our exponential BGP variant, unemployment vanishes as  $k \rightarrow 0$ , so welfare given in equation (47) simplifies to

$$\begin{aligned} \lim_{k \rightarrow 0} W &= \int_{t=0}^{\infty} e^{-rt} Z_t dt = \int_{t=0}^{\infty} e^{-rt} \left[ \frac{1}{\lambda} + R_0 + (g_A/\lambda) t \right] dt \\ &= \frac{1}{r} \left( \frac{1}{\lambda} + R_0 \right) + \frac{g_A/\lambda}{r^2}. \end{aligned}$$

Since  $R_0 \rightarrow \infty$ , welfare again diverges to  $+\infty$ .

**Conclusion.** In both the MM and exponential inspection frameworks, eliminating search frictions leads to unbounded welfare gains. This outcome stands in sharp contrast to the DMP model, where welfare gains are finite, and highlights a fundamental weakness of inspection models: they predict implausibly large benefits from removing frictions.

### 3.9 Possible Generalizations

In principle, equation (1) can be used to characterize the BGP path of reservation productivity implied by a given sample distribution, while equation (19) can be employed to reverse-engineer required parameter restrictions for  $b_t$  and/or  $k_t$ . Specifically, along a BGP, equation (1) can be written as

$$R_t = H^{-1}(\phi_1 e^{-g_A t}),$$

where  $H(R_t) = 1 - F(R_t)$  is the survival function and  $\phi_1 \equiv \frac{A_0 p(\theta)}{h_{ue}}$  is a constant. Then  $b_t$  and  $k_t$  must satisfy:

$$b_t = y_t H^{-1}(\phi_1 e^{-g_A t}) - k_t \frac{M_1(1, \theta)}{M_2(1, \theta)}.$$

Here  $\phi_1$  and  $\theta$  are endogenous but constant. A few illustrative cases are:

1. **Pareto:**  $F(z) = 1 - \left(\frac{z_l}{z}\right)^\alpha$ ,  $H(R) = \left(\frac{z_l}{R}\right)^\alpha$ ,  $H^{-1}(x) = z_l x^{-1/\alpha}$ ,  $R_t = z_l \phi_1 e^{g_A t/\alpha}$ .
2. **Exponential:**  $F(z) = 1 - e^{-\lambda z}$ ,  $H(R) = e^{-\lambda R}$ ,  $H^{-1}(x) = -(1/\lambda) \ln x$ ,  $R_t = -(1/\lambda) \ln \phi_1 + (1/\lambda) g_A t$ .

3. **Gompertz:**  $F(z) = 1 - e^{-\tau(e^{\lambda z} - 1)}$ ,  $H(R) = e^{-\tau(e^{\lambda R} - 1)}$ ,  $H^{-1}(x) = (1/\lambda) \ln(1 - \frac{1}{\tau} \ln x)$ ,  $R_t = (1/\lambda) \ln(1 - \frac{1}{\tau} \ln \phi_1 - \frac{1}{\tau} + g_A t)$ .
4. **Weibull:**  $F(z) = 1 - e^{-(\lambda R)^\tau}$ ,  $H(R) = e^{-(\lambda R)^\tau}$ ,  $H^{-1}(x) = (1/\lambda) (-\ln x)^{1/\tau}$ ,  $R_t = (1/\lambda) (-\ln \phi_1 + g_A t)^{1/\tau}$ .

The procedure just described provides BGP *candidates*. However, a complete solution requires to construct the proper initial distribution  $G_0$  using equation (26) and verifying that all BGP-defining equations are satisfied.

## 4 Biased Technological Change

This section develops an alternative to the inspection framework: a DMP model with homogeneous workers and biased technological progress in the matching function. We show that when technological change is biased and the inputs in the matching function are complements, a well-behaved limiting BGP exists. This BGP preserves the central properties of the standard DMP model under worker-augmenting technological progress: welfare gains from eliminating search frictions are finite, and unemployment vanishes when vacancies are costless.

The section proceeds in two parts. The first subsection formalizes the notion of biased technological progress in the matching function and provides a sharp characterization for the constant elasticity of substitution (CES) case. We show that, contrary to MM's claim, their results are not robust to the introduction of biased progress. In particular, the limiting growth rate of matches may converge to zero despite ongoing technological improvements, rendering their main theorem inapplicable in such cases. This analysis assumes a stationary tightness rate.

The second subsection embeds biased technological progress into the full DMP model. We show that a BGP with constant tightness emerges endogenously as a general equilibrium outcome—our main contribution. A key feature of this equilibrium is that it is necessarily inefficient, highlighting a sharp contrast between planner and market allocations.

### 4.1 Biased Technological Change in the Matching Function

Section 2 assumed a matching function of the form  $A_t M(u_t, v_t)$ . In this formulation, technological progress is Hicks-neutral. We now consider a more general specification,

$$M(A_t u_t, B_t v_t),$$

where  $A_t$  and  $B_t$  represent unemployment- and vacancy-augmenting technologies, growing at constant exogenous rates  $g_A \geq 0$ , and  $g_B \geq 0$ , respectively.

The job-finding rate is defined as

$$m_t \equiv \frac{M(A_t u_t, B_t v_t)}{u_t} = M(A_t, B_t \theta_t) =: m_t(\theta_t). \quad (49)$$

where  $\theta \equiv \frac{v}{u}$  is the market tightness. For later purposes, it is convenient to define effective tightness as

$$\widehat{\theta}_t \equiv \frac{B_t v_t}{A_t u_t} = \theta_t \frac{B_t}{A_t}.$$

When  $A_t = B_t$ , the function simplifies to  $A_t M(u_t, v_t)$ , the Hicks-neutral case considered by MM. MM argue that their results extend beyond Hicks-neutral progress:

”In the case of input-augmenting search progress, the rate  $g_m$  converges to some  $g_m^*$ ... In the limit as  $g_m \rightarrow g_m^*$ , our theorems hold with  $g_m^*$  replacing  $g_A$ .” (MM, footnote 10)<sup>3</sup>.

However, for their results to hold, it is essential that  $g_m^* > 0$ . Otherwise, their main results do not apply. If  $g_m = 0$ , then a Pareto distribution cannot be derived from their equation (10). Moreover, with  $g_A = 0$  and their assumption  $\delta = 0$ , job destruction disappears, and unemployment vanishes in the limit.

To see why  $g_m^* = 0$  may naturally arise under biased technological progress, consider the CES matching function:

$$M(Au, Bv) = \begin{cases} (\alpha (Au)^\sigma + (1 - \alpha) (Bv)^\sigma)^{1/\sigma}, & \sigma \leq 1, \sigma \neq 0. \\ (Au)^\alpha (Bv)^{1-\alpha} & \text{if } \sigma = 0. \end{cases} \quad (50)$$

This specification has a long tradition in the search-and-matching literature (e.g., Den Haan et al., 2000; Hagedorn and Manovskii, 2008; Petrosky-Nadeau et al., 2018). The Cobb-Douglas case corresponds to  $\sigma = 0$ . Workers and vacancies are complements if  $\sigma < 0$  and substitutes if  $\sigma > 0$ . Córdoba et al. (2024) discusses several advantages of the CES function with complementarity.

For the CES function, the growth rate of meetings is

$$g_{m,t} = \mu(\widehat{\theta}_t) g_A + (1 - \mu(\widehat{\theta}_t)) g_B, \quad (51)$$

where

$$\mu(\widehat{\theta}) = \frac{\alpha}{\alpha + (1 - \alpha) \widehat{\theta}^\sigma}. \quad (52)$$

The formula confirms that when technological progress is Hicks-neutral ( $g_A = g_B$ ), we obtain  $g_{m,t} = g_A = g_B > 0$ . The next proposition characterizes the limit behavior when technological progress is either worker or vacancy augmenting.

**Proposition 4** *Suppose  $0 < \theta < \infty$ , and either (i)  $g_A > 0$  and  $g_B = 0$ ; or (ii)  $g_B > 0$  and  $g_A = 0$ . Then*

$$g_m^* = \lim_{t \rightarrow \infty} g_{m,t} = \begin{cases} \max\{g_A, g_B\} > 0 & \text{if } \sigma > 0 \\ \alpha g_A + (1 - \alpha) g_B > 0 & \text{if } \sigma = 0 \\ 0 & \text{if } \sigma < 0 \end{cases}.$$

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<sup>3</sup>MM use the notation  $g_p$  for the growth rate of meetings; we use  $g_m$ .

**Proof.** As  $t \rightarrow \infty$ , effective tightness satisfies

$$\hat{\theta}_t \rightarrow \left\{ \begin{array}{l} 0, g_A > 0, \\ \infty, g_B > 0 \end{array} \right\}.$$

From equation (52):

$$\lim_{t \rightarrow \infty} \mu(\hat{\theta}_t) = \left\{ \begin{array}{ccc} \sigma > 0 & \sigma < 0 & \\ g_A > 0 & 1 & 0 \\ g_B > 0 & 0 & 1 \end{array} \right\}.$$

Substituting into equation (51) yields:

$$\lim_{t \rightarrow \infty} g_{m,t} = \left\{ \begin{array}{ccc} \sigma > 0 & \sigma < 0 & \\ g_A > 0 & g_A & 0 \\ g_B > 0 & g_B & 0 \end{array} \right\}.$$

■

The proposition shows that when inputs are substitutes,  $g_m^* > 0$ , which is necessary for MM's results to hold. The more interesting cases are when inputs are complements ( $\sigma < 0$ ) and technological progress is biased, either worker-augmenting or vacancy-augmenting, in which cases  $g_m^* = 0$ . In these cases, MM's results do not hold. The reason is that biased progress runs into diminishing returns: the non-improving input becomes a bottleneck under strict complementarity, creating an upper bound on meetings even in the presence of continued technological progress.

We now explore the implications of this limit behavior within the DMP search-and-matching model with declining search frictions.

## 4.2 DMP Model with Biased Technological Change

There is a unit one of workers, of which  $n_t$  are employed and  $u_t$  are unemployed. Employed workers produce  $y_t = ye^{gt}$ , the unemployed workers produce  $b_t = be^{gt}$ , where  $y > b$ , and vacancy posting costs  $k_t = ke^{gt}$  at time  $t$ . Given an initial employment level  $n_0$ , the social planner solves

$$\max_{\{n_t, v_t, u_t\}_{t=0}^{\infty}} \int_0^{\infty} e^{-(r-g)t} (n_t y + u_t b - k v_t) dt \text{ subject to}$$

$$\dot{n}_t = M(A_t u_t, B_t v_t) - \delta n_t, \forall t \geq 0, \quad (53a)$$

$$u_t = 1 - n_t, \forall t \geq 0, \quad (53b)$$

where  $A_t = A_0 e^{g_A t}$  and  $B_t = B_0 e^{g_B t}$ ,  $r > g$ , and  $\delta > 0$ . Let  $\rho \equiv r - g$  denote the effective discount rate.

**Optimality conditions.** Let  $e^{-\rho t} \lambda_t$  and  $e^{-\rho t} \eta_t$  denote the Lagrange multipliers associated with constraints (53a) and (53b), respectively. The first-order conditions with respect to  $v_t$ ,  $u_t$ , and  $n_t$  are:

$$k = \frac{\partial M_t}{\partial v_t} \lambda_t = s_t (1 - \mu_t) \lambda_t, \quad (54)$$

$$\eta_t = b + \frac{\partial M_t}{\partial u_t} \lambda_t = b + m_t \mu_t \lambda_t, \text{ and} \quad (55)$$

$$y - \eta_t - \delta \lambda_t = \rho \lambda_t - \dot{\lambda}_t. \quad (56)$$

Here,  $s_t$  is the job-filling rate and  $\mu_t$  the elasticity of the matching function with respect to "effective" job seekers:

$$s_t \equiv \frac{M_t}{v_t} = m_t (\theta_t) / \theta_t \text{ and} \quad (57)$$

$$\mu_t \equiv \frac{\partial M_t}{\partial (A_t u_t)} \frac{A_t u_t}{M_t} = \mu(\hat{\theta}_t).$$

Optimality also requires the transversality condition  $\lim_{t \rightarrow \infty} e^{-\rho t} \lambda_t n_t = 0$ .

Equations (54) to (56) mirror those of the canonical DMP framework. From equation (54), the efficient number of vacancies equates the marginal cost  $k$  with the marginal new matches associated to a vacancy,  $\frac{\partial M_t}{\partial v_t}$ , multiplied by the shadow value of a match,  $\lambda_t$ . The marginal gain  $\frac{\partial M_t}{\partial v_t}$  equals the average gain  $s_t = \frac{M_t}{v_t}$ , scaled down by the elasticity  $1 - \mu_t$ .

Equation (55) states that the shadow value flow of an unemployed worker,  $\eta_t$ , is equal the worker's own output,  $b$ , plus the expected contribution to new matches,  $\frac{\partial M_t}{\partial u_t}$ , weighted by  $\lambda_t$ . Here, the marginal gain  $\frac{\partial M_t}{\partial u_t}$  equals the average gain,  $m_t$ , scaled by  $\mu_t$ .

Finally, combining equation (55) and equation (56) yields the value of a match:

$$\rho \lambda_t = y - b - (\delta + m_t \mu_t) \lambda_t + \dot{\lambda}_t. \quad (58)$$

This expression shows that the net return of a match is the added output  $y - b$  plus the capital gains,  $\dot{\lambda}_t$ , offset by the effective depreciation rate. Depreciation includes both the exogenous job destruction rate,  $\delta$ , and the endogenous effect  $m_t \mu_t$ , which captures the fact that a successful match reduces the pool of job seekers and thereby lowers future matching opportunities.

#### 4.2.1 Balanced Growth Characterization

Consider balanced growth paths (BGPs) along which variables grow at constant rates. The following proposition shows that—despite improvements in the matching technology—labor-market variables remain stationary along any BGP.

**Proposition 5** *Along a balanced growth path, the growth rates of  $n$ ,  $u$ ,  $m$ ,  $\mu$ ,  $\lambda$ ,  $s$ , and  $\theta$  are zero.*

**Proof.** *Since population is constant, both employment ( $n$ ) and unemployment ( $u$ ) must be constant along a BGP. Equation (53a) then reduces to*

$$\delta n = m_t u = m u.$$

*Thus  $m_t = m$  along a BGP. Similarly, since  $\mu_t \in [0, 1]$ , we must have  $\mu_t = \mu$ . Substituting these results into equation (58) and imposing the transversality condition yields*

$$\lambda_t = \lambda = \frac{y - b}{\rho + \delta + m\mu}.$$

*Substituting into equation (54) gives*

$$k = \frac{s_t (1 - \mu)}{\rho + \delta + m\mu} (y - b). \quad (59)$$

*Equation (59) implies that  $s_t = s$ , and since  $m = s\theta_t$  then  $\theta_t = \theta$  along a BGP.*

■

Equation (59) determines  $\theta$ , and the unemployment rate follows from

$$u = \frac{\delta}{m + \delta}. \quad (60)$$

**Markets.** Equation (59) parallels Pissarides (2000, Eq. 1.24) in a decentralized setting where firms post vacancies with success probability  $s^*$ , workers find jobs with probability  $\theta^* s^*$ , firms capture a fraction  $1 - \gamma$  of the match surplus, and free entry holds. In our notation:

$$k = \frac{s^* (1 - \gamma)}{\rho + \delta + m^* \gamma} (y - b).^4 \quad (61)$$

Free entry further implies that the expected net return of posting a vacancy,  $s^* (y - w^*) / k - \delta$ , equals the effective market return  $\rho$ , where  $w$  which yields the wage equation:

$$w^* = y - \frac{(\rho + \delta) k}{s^*}.$$

The market equilibrium is generally inefficient because the worker's bargaining power is fixed at a constant value  $\gamma > 0$ , while in the planner's allocation the *effective* bargaining power is variable,  $\mu(\hat{\theta}_t)$ . This permits the market equilibrium to sustain a BGP; the corresponding efficient allocation, by contrast, necessarily rules one out.

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<sup>4</sup>Pissarides (2000) assumes  $g = 0$ , unlike here.



### 4.2.2 Equilibrium

Consider first the market solution. According to equation (61), a BGP with declining search costs exists if they do not affect  $m^*$  or  $s^* = m^*/\theta^*$ . Taking time derivatives of equation (49) yields

$$\begin{aligned}\frac{\dot{m}_t^*}{m_t^*} &= M_1(A_t, B_t\theta^*) \frac{A_t}{M} \frac{\dot{A}_t}{A_t} + M_2(A_t, B_t\theta^*) \frac{\theta B_t}{M} \frac{\dot{B}_t}{B_t} \\ &= \mu(\hat{\theta}_t^*) g_A + (1 - \mu(\hat{\theta}_t^*)) g_B.\end{aligned}\quad (62)$$

All terms in this expression are non-negative. The following proposition follows naturally:

**Proposition 6** *There is no BGP in the market economy when  $g_A > 0$  and  $g_B > 0$ . In particular, there is no BGP with Hicks-neutral technological progress in the matching function.*

This proposition confirms MM's result for the Hicks-neutral case, namely that there is no BGP in a model without inspection. The next lemma suggests that a BGP may exist when technological change is either vacancy or worker augmenting.

**Lemma 2** Suppose  $\theta = \theta^* \in (0, \infty)$ . For  $\frac{\dot{m}_t^*}{m_t^*} = 0$ , one of the following two conditions must hold:

- (i)  $g_A > 0$ ,  $g_B = 0$ , and  $\mu(0) = 0$ , or
- (ii)  $g_B > 0$ ,  $g_A = 0$ , and  $\mu(\infty) = 1$ .

**Proof.** (i) If  $g_A > 0$  and  $g_B = 0$ , then  $\hat{\theta}_t^* \rightarrow 0$  and  $\mu(\hat{\theta}_t^*) \rightarrow \mu(0) = 0$ .

Hence,  $\frac{\dot{m}_t^*}{m_t^*} = \mu(\hat{\theta}_t^*) g_A \rightarrow 0$ . (ii) If  $g_A = 0$  and  $g_B > 1$ , then  $\hat{\theta}_t^* \rightarrow \infty$  and  $\mu(\hat{\theta}_t^*) \rightarrow \mu(\infty) = 1$ . Hence,  $\frac{\dot{m}_t^*}{m_t^*} = (1 - \mu(\hat{\theta}_t^*)) g_B \rightarrow 0$ . ■

At this point, it is convenient to focus on the CES matching function given in equation (50). Applying the lemma, we find that strict complementarity is a necessary condition for the existence of a BGP. In the CES case, the function  $\mu(\hat{\theta})$  satisfies equation (52). When inputs are strict complements,  $\sigma < 0$ , we have  $\mu(0) = 0$  and  $\mu(\infty) = 1$ , exactly as required by the lemma. In contrast, when inputs are substitutes ( $\sigma > 0$ ), we obtain  $\mu(0) = 1$  and  $\mu(\infty) = 0$ —the opposite of the condition required by the lemma. Therefore, strict complementarity is necessary, though not sufficient, for the existence of a BGP.

### 4.3 CES Matching

**Proposition 7** *Suppose  $M$  is a CES matching function. An interior BGP of the market economy exists in the following two cases:*

(i)  $g_A > 0$ ,  $g_B = 0$ ,  $\sigma < 0$ ,  $\gamma > 0$ , and

$$k < \frac{(1-\alpha)^{1/\sigma} B (1-\gamma)}{\rho + \delta} (y - b); \quad (63)$$

(ii)  $g_A = 0$ ,  $g_B > 0$ ,  $\sigma < 0$ ,  $\gamma < 1$ , and  $y > b$ .

**Proof.** (i) Under the stated conditions, the matching function converges to  $M(Au, Bv) = (1-\alpha)^{1/\sigma} Bv$ . Hence,  $s_1^* = (1-\alpha)^{1/\sigma} B$ ,  $m_1^* = (1-\alpha)^{1/\sigma} B\theta_1^*$ ,  $u_1^* = \frac{\delta}{m_1^* + \delta}$ ,

$$\theta_1^* = \frac{(1-\alpha)^{1/\sigma} B (1-\gamma) (y-b) - (\rho + \delta) k}{(1-\alpha)^{1/\sigma} B \gamma k}, \text{ and}$$

$$w_1^* = y - \frac{(\rho + \delta) k}{(1-\alpha)^{1/\sigma} B}.$$

Condition (63) guarantees that an interior solution for  $\theta^*$  exists. (ii) Under the stated conditions, the matching function converges to  $M(Au, Bv) = \alpha^{1/\sigma} Au$ . Thus,  $s_2^* = \alpha^{1/\sigma} A/\theta_2^*$ ,  $m_2^* = \alpha^{1/\sigma} A$ ,  $u_2^* = \frac{\delta}{m_2^* + \delta}$ ,

$$\theta_2^* = \frac{\alpha^{1/\sigma} A (1-\gamma)}{\rho + \delta + \alpha^{1/\sigma} A \gamma} \frac{y-b}{k}, \text{ and}$$

$$w_2^* = y - \frac{(\rho + \delta) (1-\gamma) (y-b)}{\rho + \delta + \alpha^{1/\sigma} A \gamma}.$$

An interior solution exist iff  $0 < \gamma < 1$  and  $y > b$ . ■

**Discussion.** The limit BGP characterized in Proposition 1 emerges because the CES matching function converges to a linear technology in which the sole effective input is the one not experiencing technological progress. With labor-augmenting progress, the matching function converges to a linear function of effective vacancies. Conversely, with vacancy-augmenting progress, it converges to a linear function of effective unemployed workers.

Despite these asymptotic linearities, the unemployment rate remains well behaved. For example, increases in the vacancy posting cost, unemployment benefits, or workers' bargaining power reduce market tightness and raise unemployment in the usual way.

Proposition 7 provides a counterexample to MM's claim—made in their footnote 10—that their model remains valid in the limit even under input-specific technological change. Not only the growth rate of the meetings rate goes to zero in these cases, but the canonical DMP model delivers a well-defined limit BGP without requiring heterogeneity, Pareto distributions, or inspection.

Of the two cases identified in Proposition 7, case (i) is the only one that delivers the standard result that unemployment vanishes when vacancy posting is costless. Furthermore, Córdoba et al. (2024) also show that labor-augmenting technological progress in the matching function can account for a significant share of the decline in the labor share and the fall in market tightness observed between 1980 and 2007.

### 4.3.1 Welfare Cost of Search Frictions

An important distinction between the DMP model analyzed in this section and the inspection models discussed previously lies in the potential welfare gains from eliminating search frictions. In a BGP, social welfare satisfies:

$$\begin{aligned}
W(u) &= \frac{ny + ub - kv}{r - g} = \frac{y - uy + ub - k\theta u}{r - g} \\
&= \frac{y - (y - b) \left( \frac{\rho + \delta + m}{\rho + \delta + m\mu} \right) u}{r - g} \quad (\text{using equations (57) and (61)}) \\
&= y \frac{1 - (1 - \varphi) \left( \frac{u(\rho + \delta) + (1 - u)\delta}{u(\rho + \delta) + (1 - u)\mu\delta} \right) u}{r - g} \quad (\text{using equation (60)}),
\end{aligned}$$

where  $\varphi = b/y$ .

The relative welfare costs of search frictions can then be defined as:

$$\Psi(u) \equiv \frac{W(0) - W(u)}{W(0)} = (1 - \varphi) \left( \frac{u(\rho + \delta) + (1 - u)\delta}{u(\rho + \delta) + (1 - u)\mu\delta} \right) u.$$

This measure is relative to the ideal benchmark of full employment. Such benchmark is achieved when  $k = 0$  in case (i) of Proposition 7 but not in case (ii). The key point is that the welfare costs of search frictions, or unemployment for short, is bounded above by  $1 - \varphi$ , which occurs when  $\mu = 0$  and  $\rho = 0$ .

In practice, estimated welfare costs fall well below this upper bound. For example, under the parametrization employed by Shimer (2005), the welfare cost is  $\Psi(u) = 6.5\%$ .<sup>5</sup>

## 5 Conclusion

Martellini and Menzio (2020) pose a fundamental puzzle: how can technological progress in the matching function (“declining search frictions”) be reconciled with the empirical stationarity of unemployment, tightness, and the Beveridge curve? In the spirit of King et al. (1988), they seek necessary and sufficient conditions under which balanced growth can coexist with those stationary labor-market facts.

This paper offers three main conclusions.

First, MM’s characterization is too strong. Their conditions (inspection goods with Pareto-distributed quality) are sufficient but not necessary. Balanced growth paths arise outside their framework.

Second, the inspection approach has implausible implications in the cases we study. In the Pareto version, unemployment persists even when vacancies are free to post, and the welfare gains from eliminating search frictions are unbounded. In the exponential version, unemployment does vanish with costless posting, yet the welfare gains remain infinite because workers keep raising

<sup>5</sup>We use  $\varphi = 0.4$ ,  $\rho = 0.012$ ,  $\mu = 0.72$ ,  $\delta = 0.1$ , and  $u = 4\%$ .

their reservation standards as matching becomes arbitrarily easy and the quality support is unbounded.

Third, a constructive alternative exists within a standard DMP environment once we allow for biased technological change and complementarity in matching. With complementary inputs, biased progress in one input makes the other input relatively scarcer, creating a bottleneck and hence diminishing returns to search improvements. The growth rate of meetings falls to zero, delivering a well-behaved BGP with stationary unemployment, tightness, and transition rates. In this setting, unemployment vanishes as frictions disappear but only if progress is worker-augmenting, and—crucially—welfare gains are finite. However, the BGP is necessarily inefficient: the market equilibrium admits a stationary path with declining frictions, whereas the planner’s allocation does not, reflecting the failure of the Hosios condition when bargaining weights are fixed but the planner’s shadow elasticity varies with tightness.

These results reframe the interpretations MM consider. They do not support the view that search frictions are irrelevant, nor that the historical decline in frictions has been too small. Instead, they point to a specific countervailing mechanism—endogenous bottlenecks from complementarity under biased progress—that can neutralize the growth effects of improved matching while preserving stationary labor-market variables. They also show that MM’s sufficiency result does not pin down a unique path: stationarity can emerge without perpetual growth in reservation quality (as in the exponential case) and without attributing growth to declining frictions (as in the biased-technology DMP case).

Finally, this agenda opens clear avenues for future work. Empirically, measuring the bias in the matching progress (worker- vs. vacancy-augmenting) and the degree of complementarity is central to distinguishing between inspection and bottleneck mechanisms and to conducting credible welfare assessments. Theoretically, exploring policy in environments with biased progress and complementarity—where market BGPs are inefficient—can clarify the role of bargaining institutions as well as vacancy taxes or subsidies. Relatedly, Córdoba et al. (2024) show that CES matching with worker-augmenting progress can account for secular movements in the labor share and tightness, underscoring the empirical relevance of biased technological change in matching.

## References

- Benhabib, J., J. Perla, and C. Tonetti (2021). Reconciling models of diffusion and innovation: A theory of the productivity distribution and technology frontier. *Econometrica* 89(5), 2261–2301.
- Buera, F. J. and E. Oberfield (2020). The global diffusion of ideas. *Econometrica* 88(1), 83–114.
- Córdoba, J. C., A. T. Isojärvi, and H. Li (2024). Endogenous bargaining power and declining labor compensation share. Technical report, Opportunity and Inclusive Growth Institute, Minneapolis Fed, Working Paper No. 92.

- Den Haan, W. J., G. Ramey, and J. Watson (2000). Job destruction and propagation of shocks. *American Economic Review* 90(3), 482–498.
- Hagedorn, M. and I. Manovskii (2008). The cyclical behavior of equilibrium unemployment and vacancies revisited. *American Economic Review* 98(4), 1692–1706.
- Hosios, A. J. (1990). On the efficiency of matching and related models of search and unemployment. *The Review of Economic Studies* 57(2), 279–298.
- King, R. G., C. I. Plosser, and S. T. Rebelo (1988). Production, growth and business cycles: I. the basic neoclassical model. *Journal of Monetary Economics* 21(2-3), 195–232.
- Lucas, R. E. and B. Moll (2014). Knowledge growth and the allocation of time. *Journal of Political Economy* 122(1), 1–51.
- Martellini, P. and G. Menzio (2020). Declining search frictions, unemployment, and growth. *Journal of Political Economy* 128(12), 4387–4437.
- Perla, J. and C. Tonetti (2014). Equilibrium imitation and growth. *Journal of Political Economy* 122(1), 52–76.
- Petrosky-Nadeau, N., L. Zhang, and L.-A. Kuehn (2018). Endogenous disasters. *American Economic Review* 108(8), 2212–45.
- Pissarides, C. A. (2000). *Equilibrium unemployment theory*. MIT press.
- Shimer, R. (2005). The cyclical behavior of equilibrium unemployment and vacancies. *American Economic Review* 95(1), 25–49.

## Appendix

**Proof of Equation (11):** To solve this differential equation, write it as

$$e^{-(r+\delta)\tau} \left[ \dot{\lambda}_{t+\tau}(z) - (r+\delta) \lambda_{t+\tau}(z) \right] = e^{-(r+\delta)\tau} [y_{t+\tau} R_{t+\tau} - y_{t+\tau} z].$$

Integrating yields:

$$\int_0^d e^{-(r+\delta)\tau} \left[ \dot{\lambda}_{t+\tau}(z) - (r+\delta) \lambda_{t+\tau}(z) \right] d\tau = \int_0^d e^{-(r+\delta)\tau} [y_{t+\tau} R_{t+\tau} - y_{t+\tau} z] d\tau.$$

The integral on the left-hand side simplifies to:

$$\left[ e^{-(r+\delta)\tau} \lambda_{t+\tau}(z) \right]_0^d = e^{-(r+\delta)d} \lambda_{t+d}(z) - \lambda_t(z).$$

**Proof of Equation (36):** Let  $a =: g_A/\lambda$ . Equation (13) becomes

$$\begin{aligned} \lambda_t(z) &= \int_0^{d(z,t)} e^{-(r+\delta)\tau} (z - R_t - a\tau) d\tau \\ &= (z - R_t) \int_0^{d_t(z)} e^{-(r+\delta)\tau} d\tau - a \int_0^{d_t(z)} \tau e^{-(r+\delta)\tau} d\tau \\ &= (z - R_t) \left[ -\frac{e^{-(r+\delta)\tau}}{r+\delta} \right]_0^{d_t(z)} - a \left[ -e^{-(r+\delta)\tau} \left( \frac{\tau}{r+\delta} + \frac{1}{(r+\delta)^2} \right) \right]_0^{d_t(z)} \\ &= \frac{z - R_t}{r+\delta} \left[ -e^{-(r+\delta)d_t(z)} + 1 \right] - \frac{a}{r+\delta} \left[ -e^{-(r+\delta)d_t(z)} \left( d_t(z) + \frac{1}{r+\delta} \right) + \frac{1}{r+\delta} \right] \\ &= \frac{ad_t(z)}{r+\delta} \left[ 1 - e^{-(r+\delta)d_t(z)} \right] + \frac{a}{r+\delta} \left[ e^{-(r+\delta)d_t(z)} \left( d_t(z) + \frac{1}{r+\delta} \right) - \frac{1}{r+\delta} \right] \\ &= \frac{a}{r+\delta} \left[ d_t(z) \left[ 1 - e^{-(r+\delta)d_t(z)} \right] + e^{-(r+\delta)d_t(z)} \left( d_t(z) + \frac{1}{r+\delta} \right) - \frac{1}{r+\delta} \right] \\ &= \frac{a}{r+\delta} \left[ d_t(z) \left[ 1 - e^{-(r+\delta)d_t(z)} \right] + d_t(z) e^{-(r+\delta)d_t(z)} + e^{-(r+\delta)d_t(z)} \frac{1}{r+\delta} - \frac{1}{r+\delta} \right] \\ &= \frac{a}{r+\delta} \left[ d_t(z) + \frac{e^{-(r+\delta)d_t(z)}}{r+\delta} - \frac{1}{r+\delta} \right]. \end{aligned}$$

We next need to calculate

$$\begin{aligned} \int_{R_t} \lambda_t(z) f(z) dz &= \frac{a}{r+\delta} \int_{R_t} \left[ d_t(z) + \frac{e^{-(r+\delta)d_t(z)}}{r+\delta} - \frac{1}{r+\delta} \right] f(z) dz \\ &= \frac{a}{r+\delta} \int_{R_t} \left[ \frac{z - R_t}{a} + \frac{e^{-(r+\delta)\frac{z-R_t}{a}}}{r+\delta} - \frac{1}{r+\delta} \right] \lambda e^{-\lambda z} dz \end{aligned}$$

$$\begin{aligned}
\int_{R_t} \lambda_t(z) f(z) dz &= (1 - F(R_t)) \int_{R_t} \lambda_t(z) \frac{f(z)}{1 - F(R_t)} dz \\
&= (1 - F(R_t)) E[\lambda_t(z) | z > R] \\
&= (1 - F(R_t)) \frac{a}{r + \delta} E \left[ d_t(z) + \frac{e^{-(r+\delta)d_t(z)}}{r + \delta} - \frac{1}{r + \delta} | z > R \right] \\
&= (1 - F(R_t)) \frac{a}{r + \delta} \left\{ E_{z>R} [d_t(z)] + E_{z>R} \left[ \frac{e^{-(r+\delta)d_t(z)}}{r + \delta} \right] - \frac{1}{r + \delta} \right\}.
\end{aligned}$$

Now,

$$E_{z>R} [d_t(z)] = E_{z>R} \left[ \frac{z - R_t}{a} \right] = \frac{1}{a} E_{z>R_t} z - \frac{R_t}{a} = \frac{1}{a} \left( \frac{1}{\lambda} + R_t \right) - \frac{R_t}{a} = \frac{1}{a\lambda}$$

$$\begin{aligned}
E_{z>R} \left[ \frac{e^{-(r+\delta)d_t(z)}}{r + \delta} \right] &= \frac{1}{r + \delta} E_{z>R} \left[ \frac{e^{-(r+\delta)d_t(z)}}{r + \delta} \right] \\
&= \frac{1}{r + \delta} \left[ \frac{\lambda a}{r + \delta + \lambda a} \right].
\end{aligned}$$

Therefore,

$$\begin{aligned}
\int_{R_t} \lambda_t(z) f(z) dz &= (1 - F(R_t)) \frac{a}{r + \delta} \left\{ \frac{1}{a\lambda} + \frac{1}{r + \delta} \frac{\lambda a}{r + \delta + \lambda a} - \frac{1}{r + \delta} \right\} \\
&= e^{-\lambda R_t} \frac{a}{r + \delta} \left[ \frac{1}{a\lambda} + \frac{1}{r + \delta} \frac{\lambda a}{r + \delta + \lambda a} - \frac{1}{r + \delta} \right] \\
&= e^{-\lambda R_t} \frac{1}{(r + \delta)\lambda} \left[ 1 + \frac{1}{r + \delta} \frac{(\lambda a)^2}{r + \delta + \lambda a} - \frac{a\lambda}{r + \delta} \right] \\
&= e^{-\lambda R_t} \frac{1}{(r + \delta)\lambda} \left[ \frac{r + \delta - g_A}{r + \delta} + \frac{1}{r + \delta} \frac{g_A^2}{r + \delta + g_A} \right]. \quad (64)
\end{aligned}$$

**Proof of Proposition 2:** Substituting equation (42) and equation (43) into equation (39):

$$\begin{aligned}
h_{ue}^\infty &= A_0 z_l^\alpha \left( \Omega_1 k_0^{-\frac{\alpha}{\alpha - (1-\gamma)}} \right)^{1-\gamma} \left( \frac{\gamma}{1-\gamma} \frac{\Omega_1}{y_0} k_0^{-\frac{1-\gamma}{\alpha - (1-\gamma)}} \right)^{-\alpha} \\
&= \Omega_2 k_0^{\frac{\alpha(1-\gamma) - \alpha(1-\gamma)}{\alpha - (1-\gamma)}} = \Omega_2,
\end{aligned}$$

where

$$\begin{aligned}
\Omega_2 &= A_0 z_l^\alpha \Omega_1^{1-\gamma} \left( \frac{\gamma}{1-\gamma} \frac{\Omega_1}{y_0} \right)^{-\alpha} \\
&= A_0 z_l^\alpha \left( \frac{\gamma}{1-\gamma} \frac{1}{y_0} \right)^{-\alpha} \Omega_1^{1-\gamma-\alpha} \\
&= A_0 z_l^\alpha \left( \frac{\gamma}{1-\gamma} \frac{1}{y_0} \right)^{-\alpha} \left( \frac{1-\gamma}{\gamma} (A_0 (1-\gamma) \Phi_1)^{1/(\alpha-1)} y_0^{\frac{\alpha}{\alpha-1}} \right)^{\frac{1-\gamma-\alpha}{1+\gamma/(\alpha-1)}} \\
&= A_0^{1+\frac{1}{\alpha-1} \frac{1-\gamma-\alpha}{1+\gamma/(\alpha-1)}} y_0^{\alpha+\frac{\alpha}{\alpha-1} \frac{1-\gamma-\alpha}{1+\gamma/(\alpha-1)}} \\
&\quad \times z_l^\alpha \left( \frac{\gamma}{1-\gamma} \right)^{-\alpha} \left( \frac{1-\gamma}{\gamma} ((1-\gamma) \Phi_1)^{1/(\alpha-1)} \right)^{\frac{1-\gamma-\alpha}{1+\gamma/(\alpha-1)}} \\
&= z_l^\alpha \left( \frac{\gamma}{1-\gamma} \right)^{-\alpha} \left( \gamma^{-1} (1-\gamma)^{\alpha/(\alpha-1)} \right)^{\frac{1-\gamma-\alpha}{1+\gamma/(\alpha-1)}} \Phi_1^{-1} \\
&= z_l^\alpha \gamma^{-\alpha-\frac{1-\gamma-\alpha}{1+\gamma/(\alpha-1)}} (1-\gamma)^\alpha (1-\gamma)^{-\alpha} \Phi^{-1} \\
&= z_l^\alpha \gamma^{-1} \Phi_1^{-1}.
\end{aligned}$$

Therefore,

$$\begin{aligned}
h_{UE}^\infty &= \Omega_2 = z_l^\alpha \gamma^{-1} \Phi_1^{-1} = \frac{z_l^\alpha \gamma^{-1}}{\frac{z_l^\alpha}{(\alpha-1)((\alpha-1)g_z+(r+\delta-g_y))}} \\
&= (\alpha-1) \left( \frac{\alpha-1}{\alpha} g_A + r + \delta - g_y \right) / \gamma.
\end{aligned}$$

**Proof of Proposition 3:** According to equation (48),  $W \rightarrow \infty$  as  $k \rightarrow \infty$  if  $(1-u) \frac{\alpha}{\alpha-1} > \frac{1-\gamma}{\gamma} u$  or

$$\begin{aligned}
\frac{\frac{\alpha}{\alpha-1}}{\frac{\alpha}{\alpha-1} + \frac{1-\gamma}{\gamma}} &= \frac{\frac{\alpha}{\alpha-1}}{\frac{\alpha\gamma+(1-\gamma)(\alpha-1)}{(\alpha-1)\gamma}} = \frac{\alpha\gamma}{\alpha\gamma + (1-\gamma)(\alpha-1)} \\
&> u^\infty = \frac{g_A + \delta}{g_A + \delta + h_{ue}}.
\end{aligned}$$

This simplifies to:

$$\begin{aligned}
\frac{\alpha}{\alpha-1} h_{ue}^\infty &> \frac{1-\gamma}{\gamma} (g_A + \delta) \\
\frac{\alpha}{\alpha-1} h_{ue}^\infty &= \frac{\alpha}{\gamma} (r + \delta - g_y + g_A - g_z) = \frac{\alpha}{\gamma} (g_A + \delta + r - g_y - g_z).
\end{aligned}$$

As long as  $r - g_y - g_z > 0$ , then because  $\alpha > 1$  and  $1 - \gamma < 1$

$$\frac{\alpha}{\alpha-1} h_{ue}^\infty = \frac{\alpha}{\gamma} (g_A + \delta + r - g_y - g_z) > \frac{1-\gamma}{\gamma} (g_A + \delta).$$